DOCUMENT RESUME

ED 028 637 EM 006 837

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Multimedia Instructional Programs in Mathematics--Demonstration and Experimentation. The Assimilation of New Media in the Instructional Program of a Rural School. Final Report.

Wisconsin Heights School District, Wis.; Wisconsin State Univ., Whitewater. School of Education.; Wisconsin Univ., Madison. Instructional Research Lab.

Spons Agency-Office of Education (DHEW), Washington, D.C.
Report No-NDEA-7
Pub Date Jun 66
Grant-OEG-7-59-9001-274
Note-387p.
EDRS Price MF-$1.50 HC-$19.45
Descriptors-Course Evaluation, Curriculum Research, Evaluation Techniques, *Grade 9, Instructional Programs, Learning Processes, *Material Development, *Mathematics Instruction, *Multimedia Instruction, *Programed Instruction, Programed Materials, Projection Equipment, Rural Schools, Slides, Teacher Developed Materials, Workbooks
Identifiers-*Edex Teaching System, Wisconsin Heights High School

The assimilation into an existing instructional program of both the new knowledge on learning processes and recent technological advances was attempted in this project through the integration of programed mathematics at the ninth grade level with the multimedia approach to instruction. Controlled experimentation led to several conclusions. New media can be effectively utilized within existing classroom conditions. Teachers should assist in developing and implementing new instructional programs to insure continued use of the programed units, as well as flexibility in administering the materials. If the programs are tailored to fit the ability levels of students, learning is effectively improved. The development of a multimedia program should adhere to a systematic plan which can be trial-tested with selected students, and the program should be modified accordingly. In addition to presenting the details of the development and testing of the multimedia materials, the report consolidates into an example and a checklist of procedures the experience gained during the project in the production of such materials. Twelve of the project's 24 programs are also included. (GO/MT)

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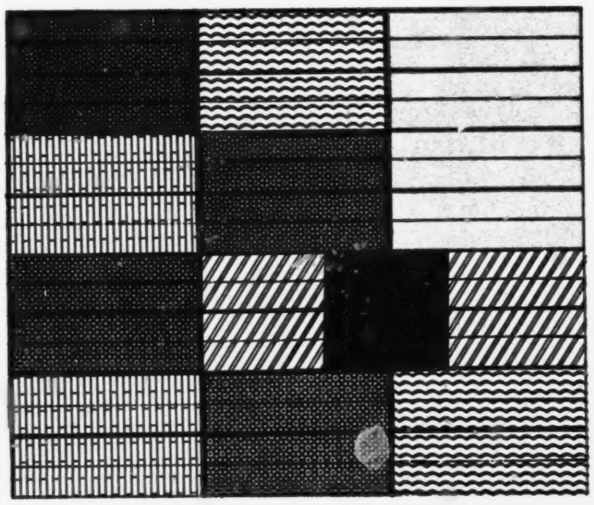
MULTIMEDIA INSTRUCTIONAL PROGRAMS IN MATHEMATICS



DEMONSTRATION AND EXPERIMENTATION

WISCONSIN HEIGHTS SCHOOL DISTRICT
THE UNIVERSITY OF WISCONSIN
WISCONSIN STATE UNIVERSITY – WHITEWATER
NDEA TITLE VII -- OE-7-59-9001-274
U.S. OFFICE OF EDUCATION

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MULTIMEDIA INSTRUCTIONAL PROGRAMS IN MATHEMATICS

-- DEMONSTRATION and EXPERIMENTATION --

The Final Report of
U. S. Office of Education Project
"The Assimilation of New Media in the
Instructional Program of a Rural School"

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The Research Reported Herein was Partially
Supported by a Grant from the
U. S. DEPARTMENT of HEALTH, EDUCATION and WELFARE
Office of Education
to the
WISCONSIN HEIGHTS SCHOOL DISTRICT
Title VII Project Number OE-7-59-9001-274
National Defense Education Act of 1958

Support and cooperation
were also provided by the

UNIVERSITY OF WISCONSIN
Instructional Research Laboratory
Wisconsin Improvement Program -- Multimedia Instructional Laboratory

WISCONSIN STATE UNIVERSITY -- WHITEWATER
School of Education

EM006837

... many shall run to and fro,

and knowledge shall be increased.

Daniel 12:4

PREFACE

An important means of improving instruction is the integration of new knowledge and resources in ways which facilitate student learning in the classroom. Research is a major strategy for empirically determining the manner in which the integration of knowledge and resources might occur for effectively improving instruction. The research work reported herein focused on such a determination with respect to the improvement of mathematics instruction in the ninth grade curriculum of a small rural high school. From this viewpoint, the project goal was to determine the most effective way in which new knowledge of learning processes might be integrated with the new technology of programed learning and multimedia instructional techniques.

Basic to this goal were three intents:

1. to combine the principles of programed learning with the technology of multimedia instruction,
2. to prepare automated instructional units in mathematics according to the combination of selected principles of programed learning and particular principles of multimedia technology, and
3. to identify the optimal learning conditions under which the prepared instructional units would permit maximal learning; the conditions refer to learner characteristics with respect to operational definitions of high and low ability in mathematics.

The three intentions determined the course of project activities and developments. The first intent involved the selection of programed learning principles which could be operationally integrated with the characteristics of the particular multimedia system used. Efforts were directed toward extensive explorations of the various modes in which the learners could respond during instruction. These modes were conditioned by the equipment and the behavioral expectations of the learner.

The second intent consisted of reviewing the existing curriculum with reference to limitations experienced in present teaching procedures and learning activities. This concern was fundamental since existing circumstances form the foundation upon which change is accomplished.

The third stated intent necessitated the plans and operations of the project to be structured according to an experimental design so as to insure:

1. systematic manipulation of the instructional conditions,
2. replicability of the manipulations,
3. protection against systematic bias due to factors of the instructional process which were not controlled within the project,
4. the systematic structuring of observations of the learning outcomes, and

5. an organized framework within which experimental results could be interpreted and inferences drawn.*

Thus, the design chosen resulted from both theoretical and practical considerations.

The realization of the three intentions called for the mobilization of a variety of resources and their coordination. This was accomplished through the talents and competencies of the team of students, teachers, administrators, and researchers who contributed to the fulfillment of the project intentions. The project staff wishes to make special acknowledgment of the institutional support and the individual assistance given by the following people:

The students of Wisconsin Heights High School
who participated in the project

The Board of Education
Wisconsin Heights School District

Mr. Robert E. Ames, Superintendent
Wisconsin Heights School District

The Faculty of the Wisconsin Heights High School

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*These points summarize some of the major reasons for using an experimental design. For a more systematic, in-depth treatment of the subject, the reader is referred to the following sources: D. T. Campbell and J. C. Stanley, "Experimental and quasi-experimental designs for research on teaching," in N. L. Grage (Ed.), _Handbook of research on teaching_, Chicago: Rand McNally, 1963; D. R. Cox, _Planning of experiments_, New York: Wiley, 1958; and R. A. Fisher, _The design of experiments_, London: Oliver and Boyd, 1935.

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The Multimedia Instructional Laboratory of the Wisconsin Improvement Program at the University of Wisconsin produced most of the visual displays used in the experimentation (see Part III). The project staff is most appreciative of this cooperation made possible by Professor John Guy Fowlkes, initiated under the supervision of Dr. L. Clinton West, and continued by Gerald F. McVey.

The Photographic Laboratory of the University of Wisconsin in cooperation with Jules M. Rosenthal prepared the movie film sequences. This service is most appreciated.

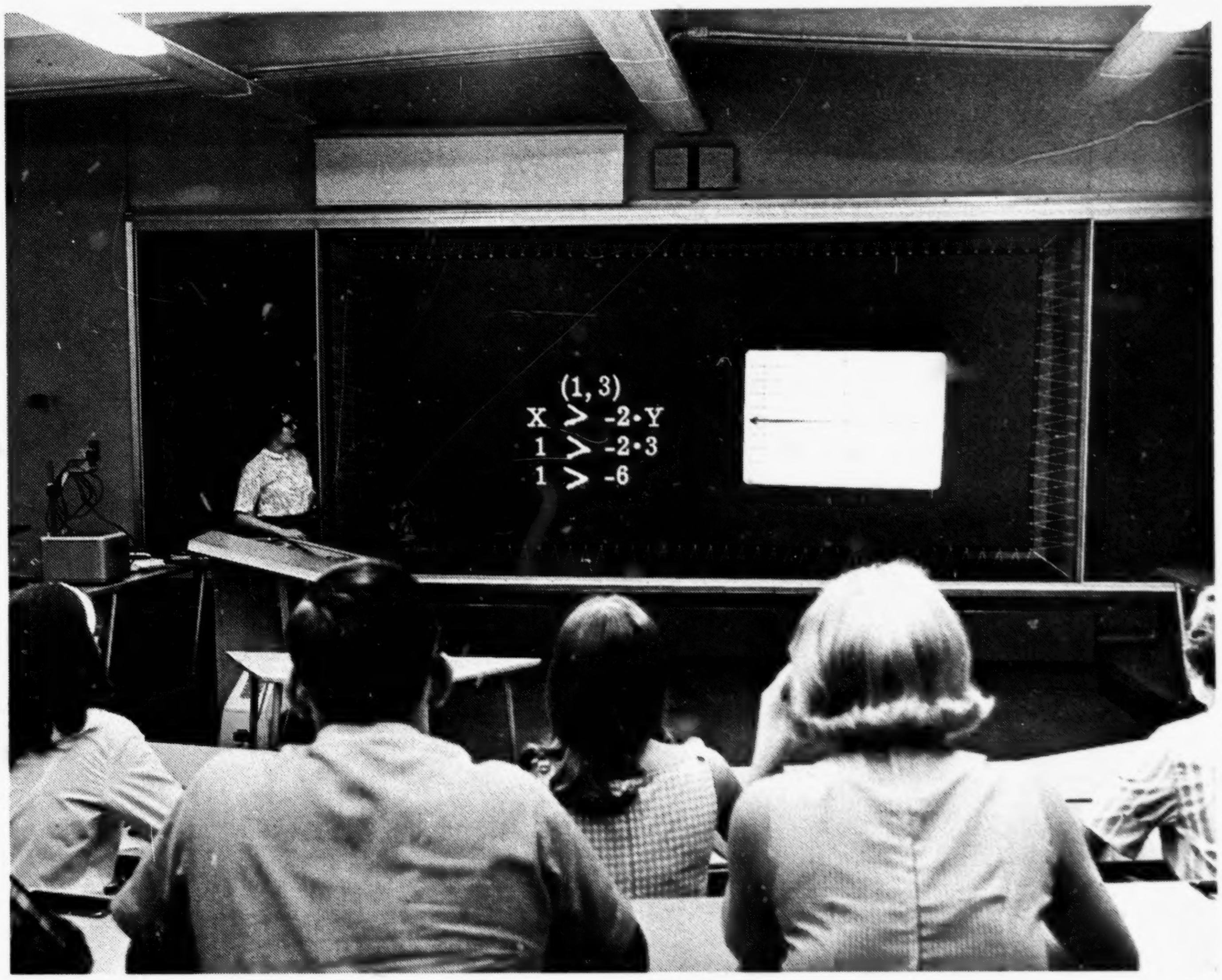
The Duplicating Service of the University of Wisconsin assisted in various ways with the final production of this report. This assistance is gratefully acknowledged.

Portions of the analyses were carried out at the Computing Center of the University of Wisconsin and this work was partially supported by the National Science Foundation and the Wisconsin Alumni Research Foundation through the University Research Committee of the University of Wisconsin.

This final report has been organized and prepared in three major parts corresponding to the three major areas of project activities. Part I, "Plans, Experiments and Outcomes", presents the details of the research developments, the experimental results, the conclusions, and the recommendations. Part II, "The Development of Programed Multimedia Instructional Units: A Checklist and an Example", consolidates the knowledge and experience gained during the project with respect to the production of multimedia programs for the classroom. Part III, "Selected Programs", presents twelve of the twenty-four classroom instructional programs developed for the experimentation.

D.M.M.
B.G.A.
C.A.C.
R.F.C.
L.D.M.
P.E.M.

June, 1966
Wisconsin Heights
Wisconsin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1 > -2•3
1 > -6

Classroom instruction using multimedia facilities

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PART I

PLANS, EXPERIMENTS, AND OUTCOMES

Chapter I

The Theory, Intent, and Unique Aspects of the Project

The Problem

Recent technological advances for the field of education are suggestive of the tremendous breadth the school instructional repertoire will take within the forseeable future. The introduction of these methods and techniques into the existing repertoire implies a two-phase process of assimilation for public educational institutions. It is not intended that the phases be regarded as independent, for the areas of overlap, e.g., staff utilization, seem obvious.

The first phase is the recognition of the increasing need for the new technology. Evidence of this need is reflected in several ways. First, the number of pupils attending public institutions is increasing, as is the length of time these pupils are enrolled. Secondly, the knowledge explosion has resulted in a greater amount of material to be acquired and retained by these pupils. Concurrent with these increases, there has been a decrease in the number of fully qualified, competent, and professional instructional personnel; thus, the skills these persons have must be used optimally.

The second phase focuses on the evaluation of established school practices to insure the effective implementation of the new instructional methods. Changes emanating from these evaluative procedures will involve curricular programs and the conditions under which the teaching-learning process occurs, as well as established patterns of instructional plans and staff assignments.

This assimilation process presents a major problem to an educational institution when no guidelines are available. The purpose of the present project was to study the assimilation of one automated multimedia teaching procedure, an Edex Teaching System into the instructional repertoire of a rural high school. This goal was achieved through the following steps:

1. the involvement and participation of members of the school staff;
2. the development of teaching programs appropriate for the system, and
3. the experimental study of characteristics of the teaching-learning process as conditioned by the system; this step involves the production of instructional programs for student groups differing in learning abilities.

Related Literature

Assimilating New Instructional Technology

The introduction of new instructional technology into established classroom procedures has long been a problem. Anderson (1962), in a skeletal review of the problem from 1650 to 1900, made clear the difficulties encountered by innovators of even the most simple mechanical devices. In this century, Pressey's punchboard devices received little attention until 1950 (Fattu, 1960; Lumsdaine and Glaser, 1960).

Within the last three decades, acceptance of the automated teaching procedures has shown a slow, but steady growth. During World War II, for example, a number of training devices were developed and successfully applied; these demonstrations gave impetus to the idea that automated methods could be used effectively and efficiently in a variety of educational contexts. The increasing recognition of the potential role played by multimedia instructional methods in improving educational practices is reflected in a number of publications (Finch, 1962; Gagne, 1962; Glaser, 1962; Schramm, 1964). However, personnel in public educational systems, i.e., the practitioners, have been slow to implement the new technology.

Reasons for this resistance are many and include insufficient time, lack of technical skills, insufficient funds, and inadequate facilities. The truth or falsity of these reasons is not questioned; rather, the important concern seems to lie in the identification of those conditions which facilitate and promote acceptance of new procedures as plausible alternatives to existing practices. Within the area of social psychology, some investigators have focused on the processes characterizing resistance to change; among the processes cited by Secord and Backman (1964) are selective exposure, motivational resistance to changing certain attitudes, and immunization of beliefs. More specifically, the results obtained by Coch and French (1948) and Lewin (1958) have led to the formulation of those conditions necessary for the successful implementation of innovations into established institutional practices. The conditions relevant to the present project are the following: a) the involvement of all related persons; and b) the appropriate active participation of all related personnel in the actual process of considering, planning, and executing change.

Thus, the literature would suggest, with regard to the present project, that a) the administrative and teaching staff of a public school be acquainted with programed instructional and multimedia systems; and b) the teaching staff be active participants in the development of instructional materials, the design of the teaching programs, arrangement of the multimedia equipment, and administration of experimental plans.

Empirical Research on Media-conditioned Instruction

Many studies are and have been directed toward the accumulation of objective and reliable knowledge concerning characteristics of the teaching-learning process as conditioned by automated instruction procedures. For example, a symposium (Lumsdaine, 1961) focused on the Air Force's longitudinal research on training procedures; flash cards, drill machines, recording devices, and films were used to investigate relevant variables, *e.g.*, retention, transfer, reinforcement, and feedback. Other studies have appeared in the publications of Galanter (1959), Lumsdaine and Glaser (1960) and Schramm (1964). In general, the results obtained suggest that automated teaching procedures have great potential for directing and guiding student behavior in such a way that the effectiveness of instruction is maximized.

The further individualization of instruction is one of the most promising areas emanating from the research to date. As opposed to assuming that the same procedures and presentations must serve all learners, it is certainly possible to foresee the adaptation of these instructional methods to individuals and individual groups, *e.g.*, homogeneous. Carr (1960) expressed a hope that individual differences might be reduced by programmed instruction. This general hypothesis has received partial support (Ferster and Sapon, 1958, Porter, 1959); however, no definitive conclusions can be made, for some studies report main effects due to factors of ability and intelligence (Lambert, Miller, and Wiley, 1962; Ripple, 1963).

Of greater significance for teachers is the potential of automated teaching methods for providing controlled and flexible instructional sequences suited to the learning needs of individuals and groups. Brown and Abell (1965) presented a list of 53 unanswered questions solicited from a group of researchers in mathematics education and considered to have far-reaching ramifications; several focused on the needs for flexible, individualized procedures. Specific projects concerned with individualization have included a provocative series using training films (Maccoby and Sheffield, 1958; Margolius and Sheffield, 1961; Weiss, Maccoby, and Sheffield, 1961); the investigators have concluded that greater learning is associated with a gradual increase in the length of film sequences between practice exercises than learning from exercises spaced by equal lengths of film.

Thus, the researchable areas concerned with the teaching-learning process are many and varied. Also, many specific questions concerning the process under conditions are indicated, and the relevance of the present empirical investigation is borne out.

Unique Aspects of the Project

The present project was unique in the following respects:

1. in many instances, multimedia procedures have been introduced with subjective observations of the conditioned effects; this study has obtained objective information concerning various aspects of the teaching-learning process as conditioned by the media;
2. staff members were actively engaged in phases of the development of instructional materials;
3. the project was conducted in a school district which is average in terms of size and available resources; however, the district

had the foresight to establish, by consolidation, a unified efficient, and economical institution capable of a) adapting to present and future educational needs, and b) absorbing necessary changes in that adaptation;

4. guidelines have been established to facilitate the assimilation process in other educational institutions;
5. through the cooperation of the school administration the procedural aspects of the experimentation, the project was not regarded as an overly disruptive element; on several occasions, priority was awarded to other activities, such as in-service meetings, track meets, and band rehearsals; thus, the status of the project was similar to that of academic subjects in the school curriculum;
6. experimentation was conducted within the framework of a developmental demonstration project; that is, allowance was made for revision procedures while prescribed standards of research procedures were followed;
7. the design of the experimental portion of the project permitted the acquisition of a maximal amount of information from a relatively small number of subjects;
8. most linear media-oriented programs written for a particular content area, *e.g.*, the Pythagorean theorem in plane geometry, assume that one program can serve all ability levels within the student group; in the experimental portion of the present project, each program was developed for two ability levels: high and low.

Chapter II

Program Development: The Setting and Procedures

The Setting

In 1964, consolidation agreements between the towns of Black Earth and Mazomanie resulted in the establishment of a unified, efficient, and economical institution--the Wisconsin Heights High School. This facility, which has a capacity of 500 students, currently serves approximately 400 secondary school students from those towns and the surrounding rural area. The present project was conducted in the context of this school.

An understanding of the setting is facilitated best by consideration of the instructional orientation of the school staff, which includes approximately 35 persons. This orientation focuses on the following points:

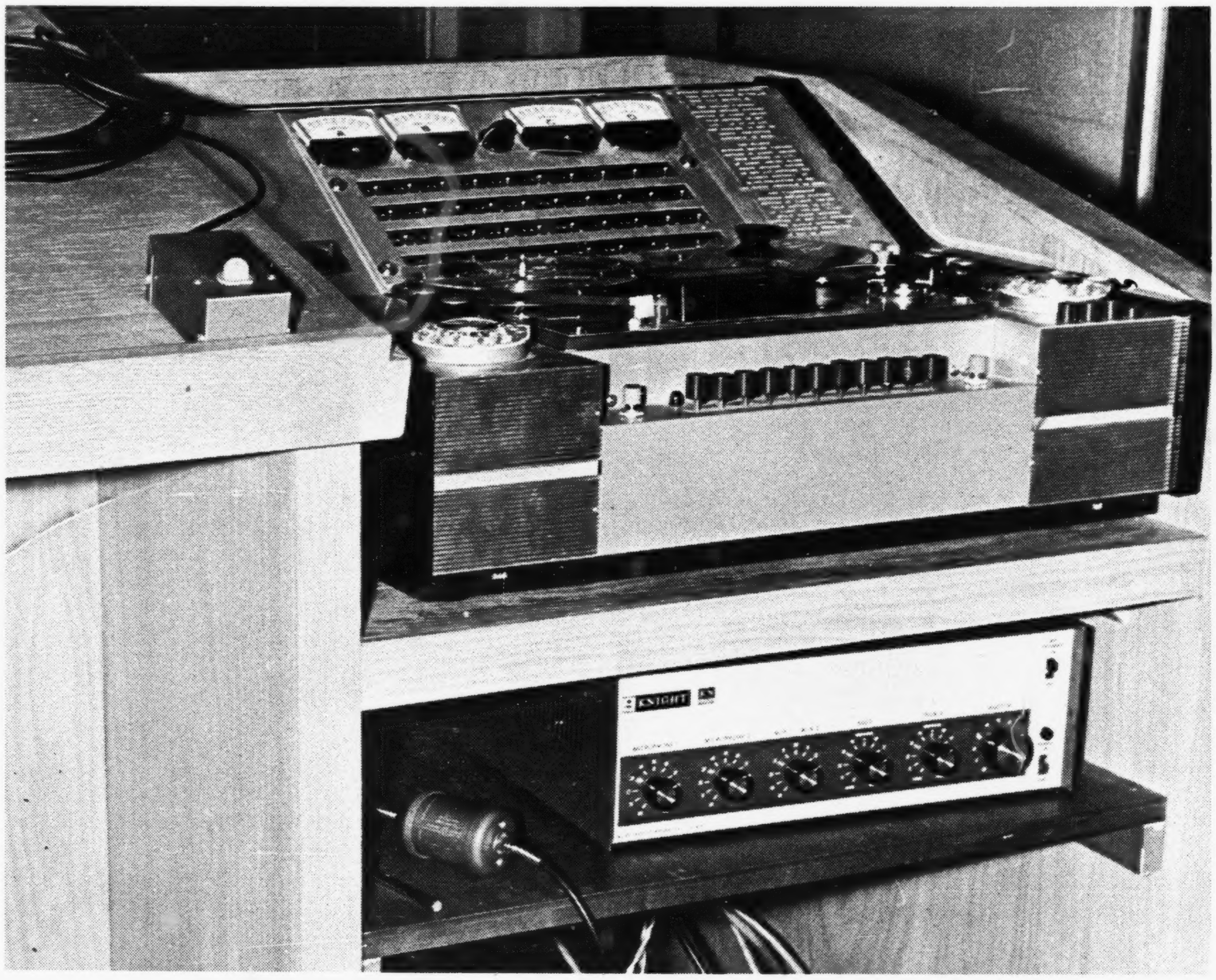
1. a combination of team teaching and seminar procedures is considered the most effective and efficient instructional approach for four academic areas: English, social studies, mathematics, and the physical sciences; thus, for a particular course offering, a teaching team plans two large-group presentations per week complemented by small-group seminars;
2. although staff members in the remaining areas are organized as teams, the more traditional "single teacher" approach is used, in general, in those areas, *e.g.*, foreign languages, home economics, and business education; and
3. independent study is regarded as the center of the educational process; implied in this point is the belief that library facilities should be extensive enough to enrich, as well as to accommodate the curricular program.

Thus, the school plant was designed and built to enhance and complement the instructional orientation. In addition to the usual services a library extends, the Instructional Materials Center (IMC) offers programed learning materials, student study carrels, conference rooms, a soundproof preview room, language and reading laboratories, teaching team meeting and preparation rooms, and teacher offices. Easily accessible, the IMC occupies the central area of the school and divides the two major instructional wings: Math-Science and Humanities (English and social studies). Each wing is dominated by a large-group lecture room accommodating over 100 students; seminar rooms holding 10 to 15 students, as well as laboratories for the sciences, and for business education, ring this central area.

Practical aspects of the daily procedures enhance the effectiveness of the Wisconsin Heights approach. For example, modular scheduling allows flexibility in the routine of both students and staff; an "honor pass" system eliminates the traditional study hall for most students and promotes the use of the IMC.

The Edex Teaching System

The facilities of the Wisconsin Heights High School include an Edex Teaching System, installed in the Humanities large-group lecture room. This device is basically a classroom communication system. A presentation or lesson is programed on one channel of a dual channel magnetic tape, while a key-pulse signal is recorded on the second channel. Thus, the dual channels control the operation of one or more pieces of standard projection equipment that present visual complements of the audio presentation. Instantaneous control of this synchronized system is incorporated into a master console, which also provides continual feedback information to the teacher by



Control console for a multimedia instructional system

Photograph 2

means of push-button response buttons at each student's position. Thus, the system enables a teacher to provide programed instruction in a multimedia setting, to make judgments concerning subsequent material based on individual and group reactions to preceding material, and to have continual feedback for the total group throughout an instructional program.

A Polaroid MP-3 camera was available for the production of slides. This ancillary device makes it possible to produce instantaneously certain types of 2 x 2 transparencies at relatively low cost.

Staff Involvement

One of the conditions cited earlier as necessary for the successful implementation of innovations into established institutional practices was the involvement of all related persons. Recognition of the need for multimedia procedures implies that the administrative and teaching staff should be acquainted with multimedia procedures. This step was achieved through staff participation in a series of in-service workshops with specialist from the University of Wisconsin (Madison and Milwaukee campuses). The series was held prior to submitting the proposal definitive of the present project.

The actual implementation of the System as a synchronized combination of media required considerable deliberation. The paucity of guidelines for implementation precluded active participation by all members of the staff and suggested that participation within a single curriculum area would be more feasible, as well as more practicable. Two criteria were determined for selection of the area: 1) the nature of the subject matter should lend itself to the frequent and effective use of programed instruction; and 2) the members of the team should express high interest in the incorporation of the Edex System into the existing repertoire. The area of mathematics was deemed most adequate in fulfilling these criteria.

Responsibility for the selection of content material and instructional resources, as well as the development and organization of the instructional programs, was assumed by the Mathematics Team Teaching Leader. This step implies the belief that the classroom teacher is the person best qualified to assess strengths and weaknesses of the existing repertoire. Resources available to the Leader in meeting these responsibilities included the following: other members of the instructional team, *i.e.*, the team teacher, team intern, teacher aide, and general aide; course outlines and published materials on programed instruction; and consultants, research assistants and special audio-visual production facilities at the University of Wisconsin.

The Early Stages of Program Development

Initially a detailed study was made of the first semester Algebra I curriculum to identify topics seemingly appropriate for multimedia programing. Eight topics were chosen to insure that four would be usable in the ensuing experimentation. The topics selected were the following: pairing of sets (2 topics); introduction to logic; an example of systems without numbers; the distributive law; a problem in ratio and proportion; an introduction to scientific notation, and a problem in relations.

Each of the eight tentatively selected areas was studied in further detail. The efficacy of programing the selected subject matter was evaluated by the preparation of instructional outlines for each topic.

These outlines included a statement of the classroom procedures usually used with each topic by the team, the student worksheets, and any audio-visual resources which typically accompany the instruction. Figure 1 shows a portion of the outline reviewing concepts necessary for understanding the distributive law.

After each instructional outline had been prepared, the content was examined to determine the section most appropriate for programing. One program unit was chosen for each of the eight topics. The first draft for each unit included a script for the audio presentation and suggestions for the accompanying visual aids. Figure 2, a portion of the Scientific Notation program, shows the relationship between the script and the visuals.

The procedures described to this point were completed within a summer. Also, it should be noted that the Team Leader had received no formal training in programing techniques; rather, her experience in the classroom was considered to be requisite for the effective selection of topics and development of programs.

Pre-experimentation Revisional Procedures

In a relatively new research area, the standards imposed by experimental research imply the necessity for pilot investigations. The presentation of pilot programs seemed advisable at this point in the project to test the program development to date and to identify unforseen difficulties.

Early in the fall semester, two topics were selected--the multiplication of binomials and sets, and programs were prepared with visual accompaniments. The team leader, one of two staff members familiar with the Edex System, programed the set lesson for classroom use with Algebra I students. This

Figure 1. A portion of an instructional outline concerning the distributive law.

STUDENT WORKSHEET

POSTUALTE 2. COMMUTATIVE LAW:

FOR EVERY PAIR OF COUNTING NUMBERS a AND b, IT IS TRUE THAT $a + b = b + a$ and $a \cdot b = b \cdot a$.

$2 + 4 = 4 + 2$

If $x + 4 = 6$ then $4 + x = 6$

If $3 + 2y = 14$ then ______ $= 14$

$3 \cdot y = y \cdot 3$

______ $= 7(x + 8)$

If $5 \cdot y = 10$ then ______ $= 10$

LECTURE ROOM PROCEDURES

Here it would be appropriate for the lecturer to describe some situations which are not commutative, such as putting on one's shoes and socks. Diagrams can be used, not so much to teach this simple idea, but to lay a foundation for their use in illustrating the distributive law.

Here it may be useful to point out that although the product of 3 X 4 is the same in each instance, the concepts are truly different. The diagrams may help illustrate this difference.

VISUALS

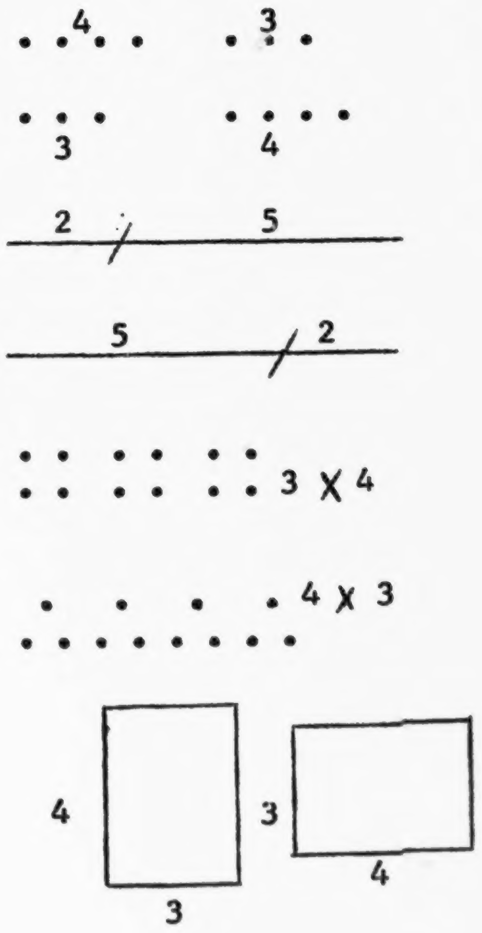


Figure 2. A portion of the first draft of the program unit concerning scientific notation

SCRIPT	VISUALS
I think all of you can see that such numerals are cumbersome to write, (3) and it seems only reasonable that any good scientist would seek to devise something better. Well a better system was evolved. A new kind of numerals came into use. These numerals are called scientific notation.	(3) Generalized picture to give impression of variety of large and small numbers. Perhaps pictures cut at odd angles and put together in random fashion a la "op art". Large and small numbers should be evident but not necessarily readable.
This notation uses exponents, so before we discuss it further let's review what we know about exponents. We have learned to write (4) a^6 when we mean a used as a factor 6 times.	(4) $a \cdot a \cdot a \cdot a \cdot a \cdot a = a^6$
In scientific notation we use powers of 10. And since it is customary to use the multiplication sign you used in grade school arithmetic, we will use it for this discussion, even though we seldom use it in algebra. 10	

Figure 2. A portion of the first draft of the program unit concerning scientific notation (contd.)

SCRIPT	VISUALS
to the sixth (5) power then would be ten used as a factor 6 times.	(5) $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$
We have also learned to use negative numbers for exponents.	
One over 10 to the sixth (6) power can be written 10 to the negative 6.	(6) $\frac{1}{10^6} = 10^{-6}$
We can see how exponential notation can be helpful in writing large numbers. For instance astronomers have found that the length of the longest observed comet is about one trillion feet. (7)	(7) (Picture of comet) underneath written 1,000,000,000,000 ft. long ----------------- 10^{12} feet long.
It would certainly be easier to write this in exponential notation.	

administration was directed toward the identification of procedural and mechanical difficulties, and several were located. First, the Edex control system did not allow the simultaneous operation of two projectors; an electronics technician was consulted, and equipment modifications were made. Second, a remote stop-start button was needed to allow the teacher some control over the program; this piece of equipment was added to the console. Third, the utility of the Polaroid MP-3 camera proved to be limited, for some of the more effective visual materials, *e.g.*, color transparencies, could not be produced; the services of the University of Wisconsin Multimedia Laboratory were obtained for the production of visuals. Finally, student reactions and technical difficulties suggested the need for further pre-experimental experiences in writing programs and preparing visuals and tape recordings.

As a result, a second pilot program was carried out in the late fall. At this time, a professional programmer was consulted through the Instructional Research Laboratory at the University of Wisconsin. This step was an integral part of the program development, for knowledge of program refinement was not presumed to be a function of the teaching staff. The distributive law program was developed in detail so that a) some assessment of the teaching-learning process could be made, and b) a model would be available for program development during experimentation.

Two revisions of the program were undertaken on the basis of the response of a few students and prior to the presentation of the program to the entire class. These revisions involved analysis of students' answers, evaluation of visuals, production of some new visuals, minor changes in the script, editing the student workbooklet, and revision of post-tests.

The introduction to the distributive law program is shown, as presented to the entire class, in Figure 3.

One procedural change resulting from the trial runs was the inclusion of a gong to indicate that the student was to work a problem in the workbooklet. Also, it was noted that the time students need to complete workbook items could not be estimated in advance; thus, it seemed advisable to use a remote stop-start button for pauses in future programs, as opposed to a programed pause. With these kinds of modifications, students' responses and technical details were considered adequate for proceeding to the experimentation.

Figure 3. The Introduction to the Distributive Law Program

SCREEN Left-hand	SCREEN Right-hand	SCRIPT
		In this lesson you will learn some elementary aspects of a new postulate. This new postulate is called the distributive law. To accomplish this, I am going to ask you to do a series of exercises which will help you understand how the law logically comes about and how some simple problems may be worked using it. Each time I ask a question or tell you to do something you will hear this gong ... You should then write the result on your worksheet.
	4 3 · 4 = 4 + 4 + 4	Please do not work ahead on the worksheet. Let us begin (4) by examining this expression. You know from elementary arithmetic that 3 times 4 means 4 + 4 + 4. Suppose the 4's are removed from this expression and in each 4's place you draw a triangle (4a). Write on your worksheet what the expression would look like.
4a 4 ------------> △		
4b 3 · △ = △ + △ + △		If the 4's are replaced by △'s the results are this. (4b)
	4c a) 3 · 4 = 4 + 4 + 4 b) 3 · (⬡+⧗) = (⬡+⧗) + (⬡+⧗) + (⬡+⧗)	Consider a and b. (4c) Parts a and b are different. The difference is that in part b each 4 has been replaced by this. (4d)
4d Each 4 is replaced by: (⬡+⧗)		

Chapter III

Experimentation:
Background Information and Methodology

Approach to the Problem

The experimental portion of this project was directed toward the assessment of characteristics of the teaching-learning process as conditioned by the Edex Teaching System. The development of the programs, including their revisions, and the design of the four experimental studies were based on the following hypothesis:

> for a given unit of subject matter, students of high and low ability (in that subject matter) will acquire and retain a maximum amount of knowledge by learning from different versions of the same instructional program.

In terms of the experimental design, which will be described in detail later in this chapter, the hypothesis was stated as follows:

> using a 2 x 2 factorial design, in which one of the factors is Ability (High and Low) and the other factor is the Program (Version A and Version B), a statistical interaction will be observed between Ability and Program.

As discussed in Chapter II, the subject matter area chosen was mathematics, and a considerable amount of program development was completed prior to the commencement of experimentation.

Definition of Terms

High ability student refers to a student achieving a score above the median of his mathematics class on the Mathematics Section of the Sequential Test of Educational Progress (STEP).

Low ability student refers to a student achieving a score below the median of his class on the Mathematics Section of the STEP.

Version A denotes a program written for high ability students.

Version B denotes a program written for low ability students.

Subjects*

Experiments 1 and 2

The subjects for the first two experiments were 62 ninth grade students enrolled in Algebra I at Wisconsin Heights High School during the 1965-66 academic year.

Experiments 3 and 4

For the third and fourth experiments, the subject group was composed of 46 sophomores and 18 juniors. All of the sophomores and one junior were enrolled in plane geometry at Wisconsin Heights High School during the 1965-66 academic year. Since the number of geometry students was not large enough to cover contingencies, e.g., absences, a list was compiled of those juniors who had enrolled in plane geometry during the 1964-65 academic year, who had not elected to enroll in trigonometry during the current year, and who had taken the STEP during their high school careers; from the resulting list of 27 juniors, 17 students were chosen at random to serve as subjects.

The mean STEP scores for the sophomore and junior groups were not significantly different. However, the results of the third and fourth experiments must be considered with caution, for the selective factors which may have differentiated between the groups were unknown and may have biased the results obtained. The nature of these selective factors may be more apparent if one considers, as an example, the composition of the groups.

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*A detailed description of student STEP performance appears in Appendix E.

On the one hand, the sophomore group was composed of students who would continue their formal mathematics training and students who would terminate their formal mathematics training upon completion of the geometry course; the junior group, on the other hand, was composed of students who had not elected a mathematics course. The psychological factors involved in the decision to continue or to terminate are complex, and the confounding effects of such factors, in general, suggest a cautious interpretation of the results.

Selection of Topics for Experimentation

During the course of the pre-experimentation program development, it became evident that the experimentation could not be conducted within the framework of the normal mathematics classroom procedures. The revision procedures considered necessary for program development required an interval of several days between administrations. Furthermore, the incorporation of revision procedures into lesson plans would have 1) broken the continuity of the presentation planned by the teaching team, or 2) required that the team give undue emphasis to a topic by extending its presentation over two or three weeks in order to subsume the experimentation. Thus, the decision was made to select topics which presumed no background beyond the skills included in the standard course syllabi, *i. e.*, first semester algebra for the freshman group and first semester geometry for the sophomore group. The topics chosen were the following:

Experiment 1: scientific notation, which served as an introduction to the topic and presumed some knowledge of exponents

Experiment 2: a system without numbers, which introduced a series of operations involving the rotation of a rectangle around various axes

Experiment 3: the graphing of inequalities, which presumed knowledge of the procedures involved in graphing a straight line

Experiment 4: linear programing, which presumed knowledge of the graphing procedures taught in Experiment 3.

A change in topic was necessitated at Experiment 3, for it was found that the students needed intensive review of linear graphing. Thus, the revisions focused on the graphing of a straight line, the original fourth topic was eliminated, and the fourth study then focused on the topic of inequalities and the number plane, i.e., the graphing of inequalities.

Program Development

Programs for the topics of the first two experiments were based on the pre-experimentation program development of the Team Leader, while the Team Intern developed the programs for the third and fourth experiments. With the aid of a professional programmer, two versions--one for high ability students and the second for low ability students--were prepared for each topic.

The approach to the preparation of each version was entirely different; that is, the program written for the high ability students was not merely a shortened version of the program written for the low ability group. Rather, the high ability program generally required that the high ability student make most of the associations necessary for learning on his own. On the other hand, the associations requisite for learning were expressed verbally in the low ability version, which required the student to answer questions about each association.

An example of this difference may be seen in the Scientific Notation program. To learn to write numbers in scientific notation, the high ability students were shown one worked example, with no verbal explanation of the procedures involved. The students were asked to observe the procedures and then to work an example in their workbook; responses to the example indicated whether correct learning associations were being made. In contrast, the version for low ability students included questions about specific associations, *e.g.*, placing decimal points, counting zeros, writing correct arithmetic symbols, and writing exponential forms.

The reader should not assume that associations were never pointed out in the Version *A* and that associations were always explained in Version *B*. The distinction between the two programs is that, *in general*, Version *A* placed the responsibility for finding the necessary associations upon the student, while Version *B* pointed to associations which were to be learned. In a word, Version *B* was considerably more directive than Version *A*.

For both versions, preparation of the Original Program (p') involved writing the script, preparing visuals, writing the student workbook, developing Measure Y_1, recording the script, and synchronizing the media; this last step was carried out, in all programs, by the Team Intern. As an aid to the programmer, a worksheet was devised and is presented in Figure 4. Also devised was a form for the specification of visuals or slides; this form is presented in Figure 5.

The Original Program, p', was revised after it had been administered once. This revision was made in light of the following evidence: an item analysis of student responses; statistical analysis, and observations made by the program administrators during the actual administration, *e.g.*, a note made of the particular time in a program when students were having difficulty understanding directions.

Figure 4. Programmer's work sheet

Page No. ______

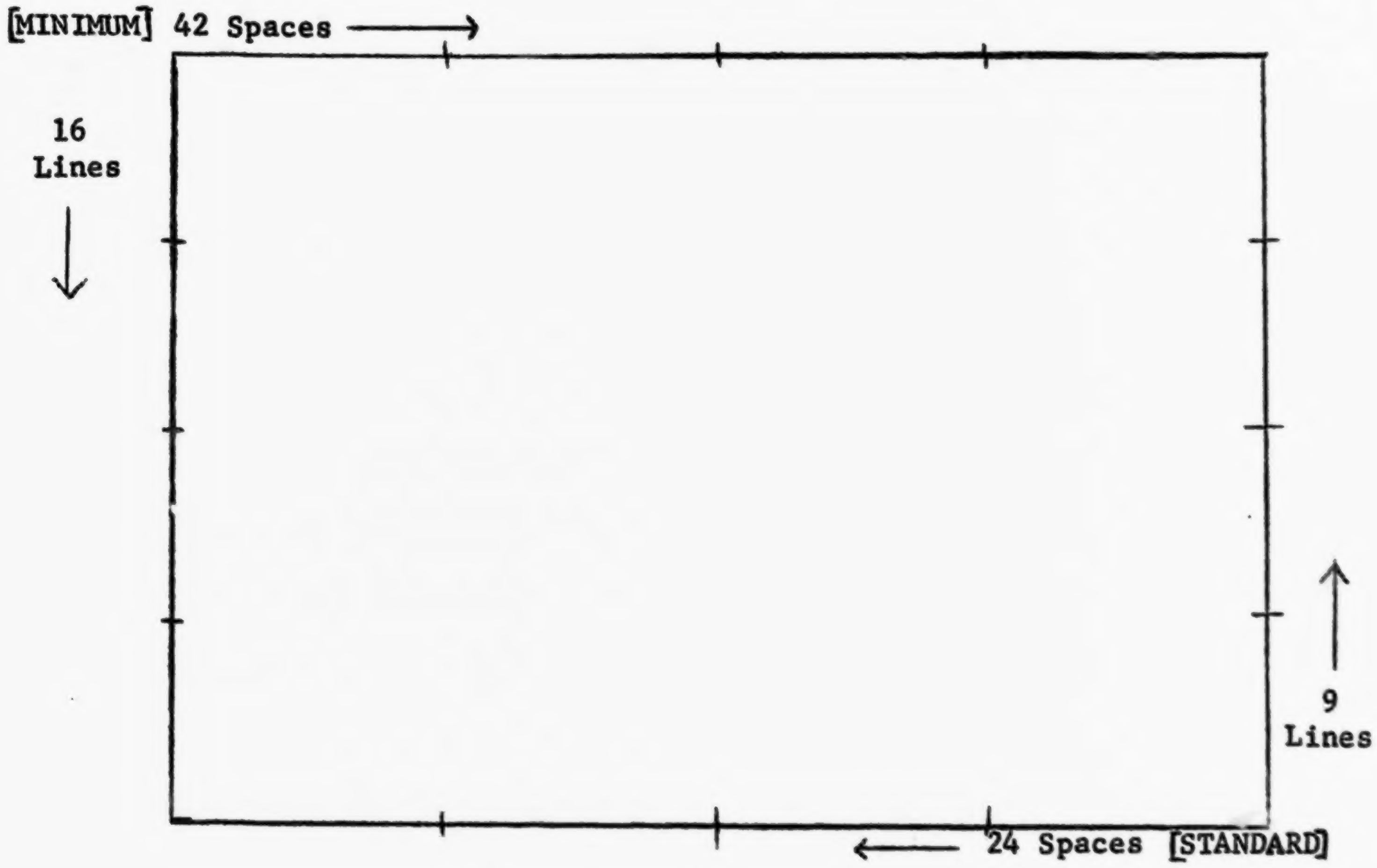
PROGRAMMING LAYOUT AND WORKSHEET

Program Title: ______________________ Date: ______________

CUE No.	SCREEN		RECORDED SCRIPT and CUE POINTS	STUDENT WORKSHEET and EXERCISES
	Left half	Right Half		

Figure 5. Slide specification sheet

Program: ____________________ Slide No. ________


[MINIMUM] 42 Spaces
16 Lines
9 Lines
24 Spaces [STANDARD]

Production Method: ______________________________

Number and letter size: 1) ____ Standard

2) ____ As large as possible

3) ____ Other ____________________
(Specify)

Special effects: ______________________________

Number of copies: ________

Review and Critique of slide (Suggested revisions):

The First Revision (p''), thus, represented changes in the script, workbooklet, and slides. These changes necessitated the preparation of a new measure, Y_2, as well as the synchronization of the revised recording and slides for actual presentation. The procedures involved in the preparation of the Second Revision (p''') and its measure (Y_3) were identical to those followed in the preparation of p''.

For a discussion of the procedures specific to a particular program, the reader is referred to Part III of this report. Therein are presented examples of programs with their corresponding measures.

Procedures

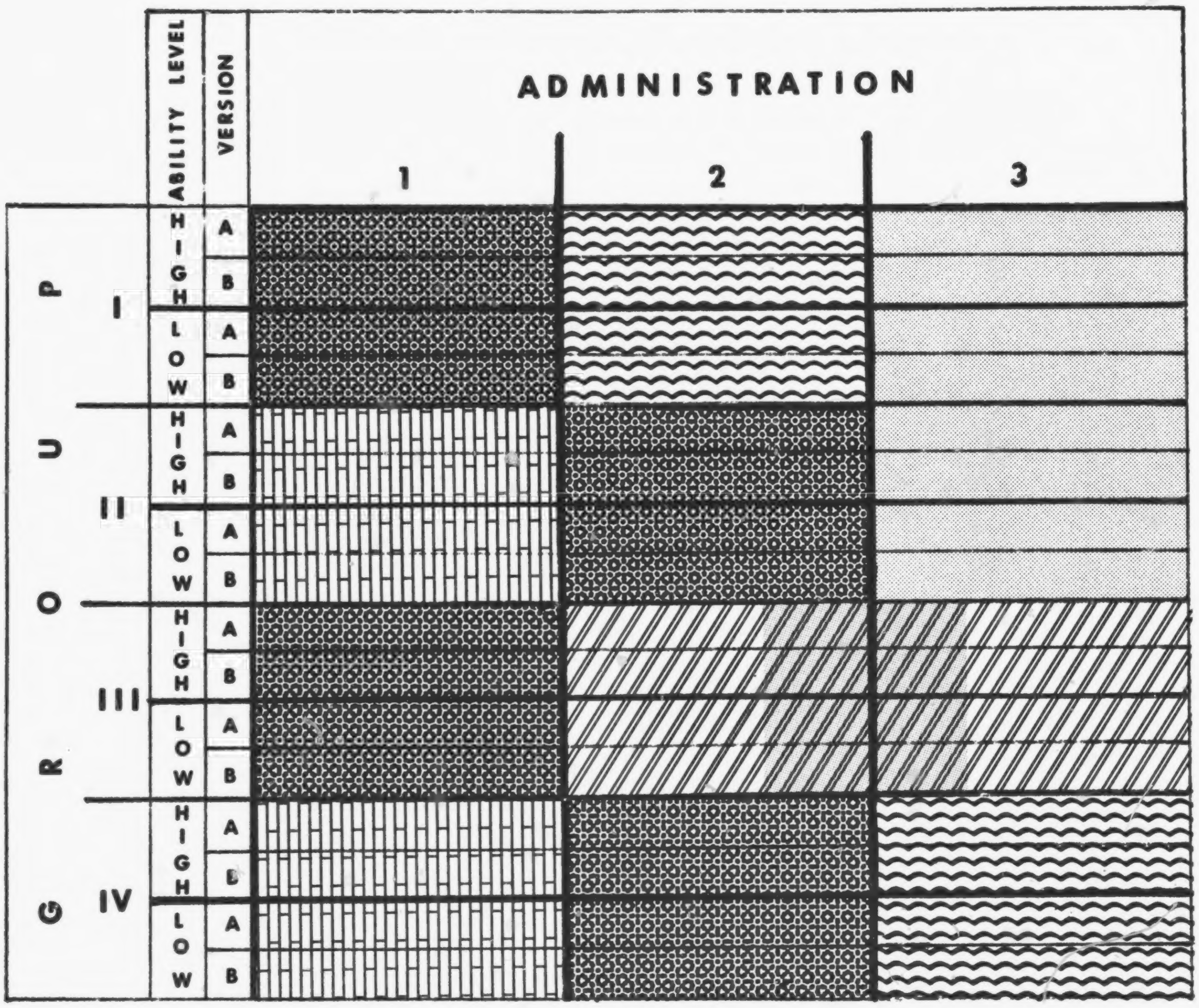
The procedures detailed below were followed for the development and experimental study of each of the four instructional programs. For the sake of clarity, the procedural descriptions have been stated in terms of the administration of one of these instructional programs.

The scientific generality of these procedures is limited to the respective population of subjects described above. The reader is reminded that the major purpose of the experimentation was to develop a method for writing and revising instructional programs, and this purpose was not undermined by the generalizability limitation imposed by the sampling techniques used.

Design

The experimental design for each of the four studies associated with the development of the versions of the four instructional programs is presented in Table 1. It can be considered as a 2^2 factorial design in which the two factors are Ability Level and Program Version. The

Table 1. Design for the experimental study of the versions of the four instructional programs



GROUP	ABILITY LEVEL	VERSION	ADMINISTRATION 1	2	3
I	HIGH	A	▓	≈	░
I	HIGH	B	▓	≈	░
I	LOW	A	▓	≈	░
I	LOW	B	▓	≈	░
II	HIGH	A	▥	▓	░
II	HIGH	B	▥	▓	░
II	LOW	A	▥	▓	░
II	LOW	B	▥	▓	░
III	HIGH	A	▓	▨	▨
III	HIGH	B	▓	▨	▨
III	LOW	A	▓	▨	▨
III	LOW	B	▓	▨	▨
IV	HIGH	A	▥	▓	≈
IV	HIGH	B	▥	▓	≈
IV	LOW	A	▥	▓	≈
IV	LOW	B	▥	▓	≈

Version: A is the program written for high ability subjects.
B is the program written for low ability subjects.

▓	Subjects view designated version of the Original Program, P', and respond to measure Y_1.
≈	Subjects view designated version of the First Revision, P'', and respond to measure Y_2.
▨	Subjects view designated version of the First Revision, P''; one week later, they respond to measure Y_2.
░	Subjects view designated version of the Second Revision, P''', and respond to measure Y_3.
▥	Subjects receive no administration.

Administration Levels 1, 2, and 3 indicate the sequence of the experimental administrations; at each level, measures of learning progress (Y_1, Y_2, and Y_3) were obtained as integral parts of the instructional programs.

Administration of the Studies

The steps for the administration of the studies, as defined by the experimental design (Table 1), are as follows:

Step 1: Subjects were randomly assigned to Groups 1, 2, 3, and 4 (as defined by the experimental design) and within the levels of ability established by the median score of the total group on the Mathematics Section of the STEP. Each group then consisted of 12 subjects and some "extras" who were designated as alternates to cover contingencies, such as absences. Each of the four groups (I, II, III, and IV) were composed, thus, of a) three low ability subjects designated to receive Version A of the Original Program (p'); b) three low ability subjects designated to receive Version B of the Original Program (p'); c) three high ability subjects designated to receive Version A of the Original Program (p'); and d) three high ability subjects designated to receive Version B of the Original Program (p').

Step 2: The versions of p' were administered to Groups I and III (Administration 1), and observations Y_1 were obtained.

Step 3: Data, obtained from Y_1, was subjected to analysis of variance and item analysis.

Step 4: On the basis of results from Step 3 and the experience of the administrators in Step 2, p' was revised, p''.

Step 5: Group I was instructed by the versions of the First Revision, p'', and observations Y_2 were obtained. Groups II and IV were instructed by the versions of p', and observations Y_1 were obtained. Group III was instructed by the versions of the First Revision, p''; no observations were obtained at this time.

Step 6: Data, Y_2 and Y_1, were subjected to analysis of variance and item analysis.

Step 7: On the basis of results from Step 6 and from the administrators' experience in Step 5, the versions of p'' were re-revised, p'''.

Step 8. Groups I and II were instructed by the versions of p''', and observations Y_3 were obtained. Measure Y_2 was administered to Group III. Group IV was instructed by the versions of p'' , and observations Y_2 were obtained.

Step 9: Data, Y_3 and Y_2, were subjected to analysis of variance.

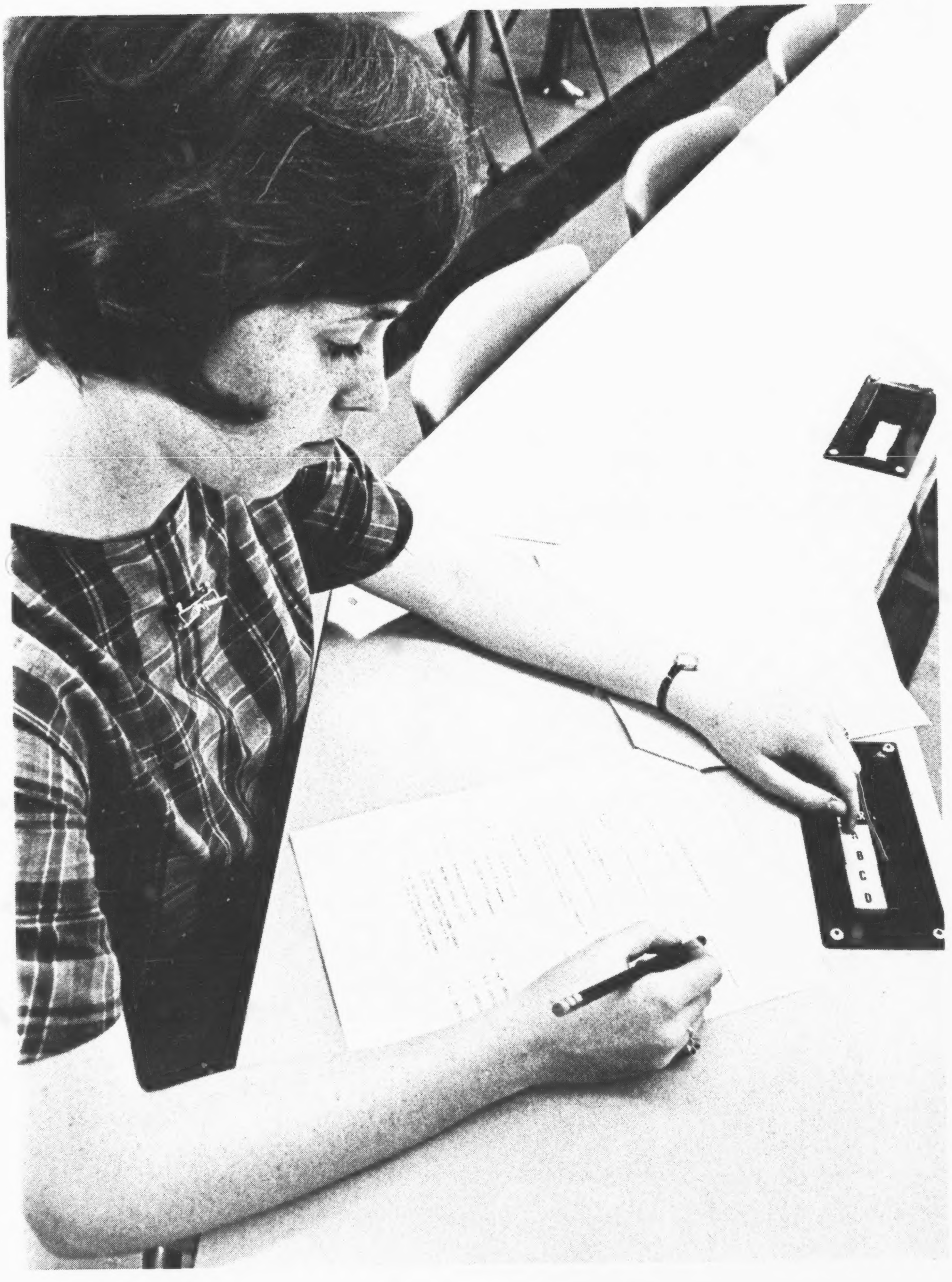
Administration of a Program

The procedures described in this section were followed in the administration of each program.

Step 1: The equipment was set up and workbooks were placed at each student's desk prior to the arrival of the students.

Step 2: One of the administrators explained the following points to the students:

1. The participation of the students was needed to assess the effectiveness of a program; that is to say, the quality of student responses would serve as an indicator of the clarity with which a concept was conveyed.
2. A brief explanation of the presentation format was given;



A student response station

Photograph 3

included was a description of the relationship among the recorded script, the visuals, and the workbook.

3. The use of a "cricket" sound effect was explained, so that the students understood that the effect was a signal to turn to a problem in the workbook.
4. The students were asked to do their best and to maintain the pace of the program, even if they had not finished a workbook item.

Step 3: The program was then begun; the tape was stopped after each "cricket" sound effect and was restarted when most of the students had finished that workbook item.

Step 4: The appropriate measure was administered; in most instances, the students were moved to another room to complete the measures so that the administration of another program could be started in the large-group room; a sufficient amount of time was allowed for each student to complete the measure.

When a problem in mechanical operation was encountered (e.g., the synchronization of slides and tapes failed, or a projector bulb burned out), the entire program was stopped until the fault was corrected; if several minutes were needed to make the correction, a short explanation was given.

A typical administration schedule may be seen by consideration of Administration 2 of the programs:

2:00 - 2:20 .... p'A
2:20 - 3:00 .... p'B
3:00 - 3:20 .... p''A
3:20 - 4:00 .... p''B.

Statistical Analysis

The statistical hypothesis under consideration stated that a statistical interaction would be observed between the Ability and Program factors. Although the experimental design suggested the plausibility of several interaction prototypes, three were considered relevant to the results.

The first relevant prototype is represented in Figure 6. The expectation in this situation was that the performance of the high ability students would not differ between versions, while the performance of low ability students would be higher on Version B. The expected lack of difference in the performance of high ability students between versions may be attributed to maximum achievement resulting from a) their high ability, and/or b) the simplicity, from the students' point of view, of the concepts to be learned.

The second prototype differs from the first with respect to the high group. Here the performance of the high ability students was expected to be lower on Version B than on Version A, as depicted in Figure 7. This difference may be considered attributable to lack of stimulation or motivation in Version B, since it seemed reasonable to assume that the high ability students should perform equally well on both versions.

The third interaction prototype is shown in Figure 8. In this case, the performance levels of both ability groups increased on Version B. This prototype could be interpreted to imply that the directive approach of Version B is required for the high ability students, as well as the low ability students.

It should be noted that the material presented in all programs hovered at a level of difficulty considerably above the low ability group. Therefore, it was expected that a ceiling effect, similar to that occurring with the

Figure 6. First interaction prototype

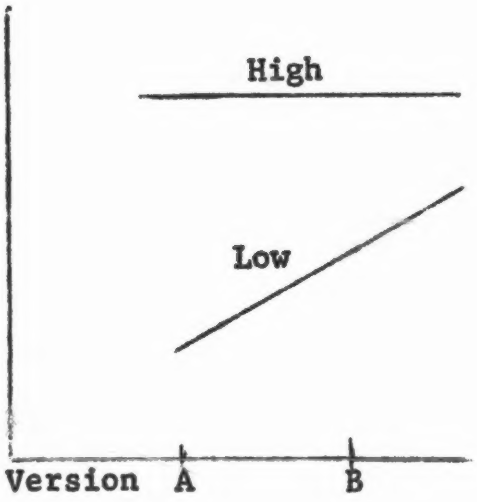


Figure 7. Second interaction prototype

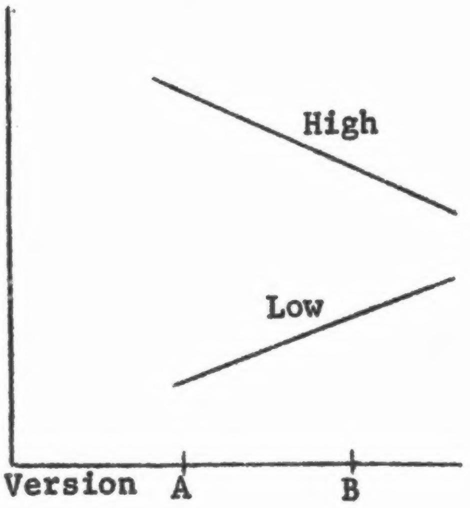
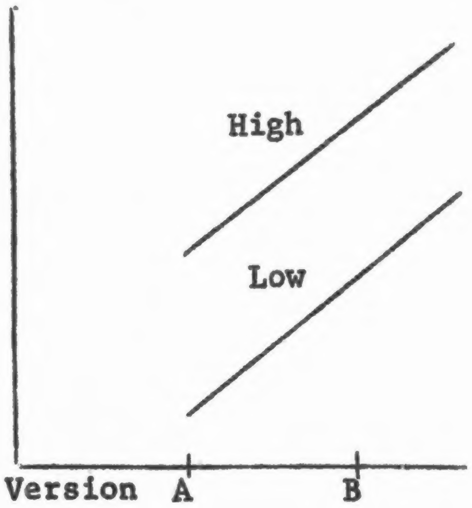


Figure 8. Third interaction prototype



high ability group in the first prototype, would not be achieved by the low ability group.

The statistical technique used to investigate the interaction of Ability and Program was the analysis of variance (Winer, 1963). The following description of the several analyses within each experiment applies to each of the four experiments, with the exceptions noted.

Two preliminary analyses were done to aid the programmer in revising p'. The first involved the pooling of the Y_1 data from Groups I and III at the first administration; the Y_1 data from Groups II and IV at the second administration were pooled for the second analysis. In each instance, the data were subjected to a 2 x 2 analysis of variance with Ability and Program as the two factors. The results of these analyses are presented in Appendix B.

Analysis 1 focused on the Y_1 data from the four groups. Like cells, e.g., high ability students who saw Version A, were combined, and a 2 x 2 analysis of variance was used with Ability and Program as the two factors.

Analysis 2 pooled the Y_2 data from Group I and the Y_1 data from Group 2 at Administration 2. Prior to pooling, the scores within each group were transformed to normalized standard scores with a mean of 50 and a standard deviation of 10, because the measures Y_1 and Y_2 could not be assumed to yield equivalent raw scores. A 2 x 2 analysis of variance was then used with Ability and Program as the two factors.

In Experiment 1, the data was considered complete enough to allow the replication of the same analysis with the Y_2 data from Group III and the Y_1 data from Group I. Thus, in that experiment this replication was called Analysis 2A.

In Analysis 3, the Y_3 data from Group II and the Y_2 data from Group IV at Administration 3 were pooled after the raw scores within each group were transformed to normalized standard scores with a mean of 50 and a standard deviation of 10. The data were then submitted to a 2 x 2 analysis of variance with Ability level and Program as the two factors. This analysis was not included in Experiment 4 because of insufficient data.

Analysis 3A followed the procedures of Analysis 3, except that the Y_3 data from Groups I and II and the Y_2 data from Groups III and IV were used. It should be noted that the Y_3 data were transformed into one distribution of normalized standard scores, and the Y_2 data into another in Analysis 3A. This analysis was feasible in Experiment 1 only.

Analysis 4 submitted the Y_3 data from Groups I and II to a 2 x 2 x 2 analysis of variance with Ability, Program, and Group as the three factors. This analysis was not included in Experiment 4 because of insufficient data.

The exploratory nature of this project indicated that trends should be noted. Thus, the means of interactions significant at the .25 level or less were investigated by means of the Newman-Keuls method (Winer, 1963) at the .05 level; the tables for these analyses appear in Appendix D.

Chapter IV

Experimentation: Results and Discussion

Results*

Experiment 1

Analysis 1. The analysis of variance of the Y_1 data revealed that 1) an interaction occurred between Ability and Program, and 2) Ability level was a statistically significant factor (.01 level), as shown in Table 2. The application of the Newman-Keuls method to the cell means, shown in Table 3, showed differences, significant at the .05 level, 1) between the high and low ability students who viewed Version A, and 2) between the high ability students who viewed Version B and the low ability students who viewed Version A. The graph of this interaction is shown in Figure 9. On the basis of these results, the following interpretations seem tenable for Experiment 1 - Analysis 1:

1. the high ability groups performed equally well on both versions; and
2. the high ability group performed better than the low ability group on the version written for the high ability group, i.e., Version A.

Analysis 2. Table 4 presents the results of the analysis of variance of the Y_2 data from Group I pooled with the Y_1 data from Group II. The main effects of Ability level and Program were significant at the .001 and .05 levels respectively. The cell means for the Ability level X Program interaction are presented in Table 5, and a graphical representation appears in Figure 10.

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*The raw data upon which the results are based appear in Appendix A.

Table 2. Experiment 1 - Analysis 1: Results of the analysis of variance of the Y_1 data

Source of Variation	SS	df	MS	F
Ability level	134.137	1	134.137	13.833[a]
Program	3.843	1	3.843	.396
Ability level X Program	15.362	1	15.362	1.584[b]
Error	504.244	52	9.697	
Total	657.586	55		

[a] significant at the .01 level
[b] significant at the .25 level

Table 3. Experiment 1 - Analysis 1: Cell means based on obtained scores

		PROGRAM VERSION	
		A	B
ABILITY	HIGH	6.413	5.619
	LOW	2.000	3.571

↔ indicates significant difference between means

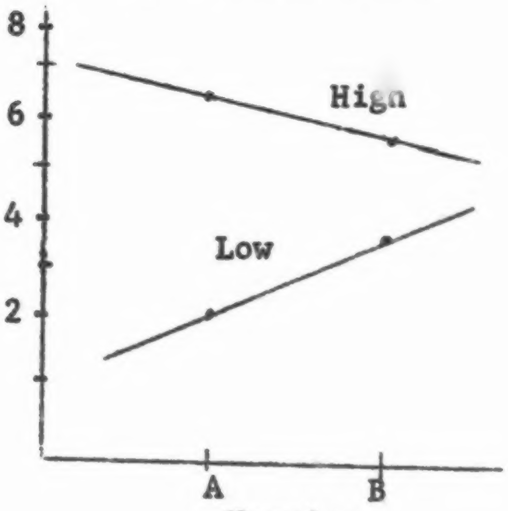


Figure 9. Graph of the Ability level X Program interaction

Table 4. Experiment 1 - Analysis 2: Results of the analysis of variance of the Group 1 (Y_2) - Group II (Y_1) data

Source of Variation	SS	df[1]	MS	F
Ability level	308.812	1	308.812	21.629[a]
Program	92.602	1	92.602	6.614[b]
Ability level X Program	37.677	1	37.677	2.691[c]
Error	1991.663	23	14.000	
Total	2424.754	26		

[a] significant at the .001 level
[b] significant at the .05 level
[c] significant at the .25 level

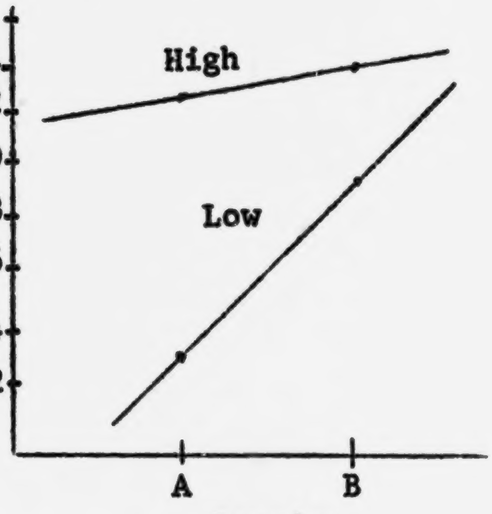
[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted when a cell entry was estimated by using the mean of the cell.

Table 5. Experiment 1 - Analysis 2: Cell means based on the pooling of normalized standard scores

ABILITY		PROGRAM VERSION A	PROGRAM VERSION B
	HIGH	52.226	53.542
	LOW	43.239	49.286

↔ indicates significant difference between means

Figure 10. Graph of the Ability level X Program interaction



The following differences were significant at the .05 level (Newman-Keuls): 1) the difference between the high and low ability students who viewed Version A; 2) the difference between the high ability students who viewed Version B and the low ability students who viewed Version A; and 3) the difference between the low ability students who viewed Version A and the low ability students who viewed Version B.

The findings in Experiment 1 - Analysis 2 suggest that:

1. the high ability students performed equally well on both versions (A and B);
2. the high ability students performed better than the low ability students on Version A;
3. the low ability group performed better on Version B, the program written for them, than did the low ability students who viewed Version A; and
4. the revision may have been a contributing factor in the appearance of Program as a significant effect at this point.

Analysis 2A. This analysis, based on the Y_2 data from Group III pooled with the Y_1 data from Group IV, was similar to Analysis 2 in the following ways: 1) Group III, like Group I, viewed p' prior to seeing p'', and 2) Group IV and Group II viewed p' together at Administration 2. However, while Group I completed Measure Y_2 immediately after viewing p'', Group III completed Measure Y_2 one week later. It would be expected that the results of Analysis 2A would approximate the results obtained in Analysis 2. However, inspection of Table 6 reveals that statistical significance was found for Ability effects in the present analysis, while Program and the interaction of Ability and Program were also significant in Analysis 2. Although other operational sources of variance, e.g., differing dependent measures, could explain this difference between analyses, it could be hypothesized that retention effects contributed to the different results obtained.

Table 6. Experiment 1 - Analysis 2A: Results of the analysis of variance of the Group III (Y_2) - Group IV (Y_1) data

Source of Variation	SS	df[1]	MS	*F*
Ability level	1490.660	1	1490.660	12.991[a]
Program	144.918	1	144.918	1.263
Ability level X Program	19.723	1	19.723	.172
Error	2639.052	23	114.741	
Total	4294.353	26		

[a] significant at the .01 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted when a cell entry was estimated by using the mean of the cell.

Analysis 3. The data involved in this analysis were obtained by pooling revision measures Y_3 and Y_2 from Groups II and IV respectively; both groups had seen p' prior to viewing a revision. Observation of Table 7 reveals that the Program effect was significant at the .01 level. Analysis of the cell means involved in the Ability level X Program interaction, as shown in Table 8, showed differences, significant at the .05 level, 1) between the high and low ability groups who viewed Version *A*, 2) between the high ability students who viewed Version *B* and the low ability students who viewed Version *A*, and 3) between the low ability students who viewed Version *A* and the low ability students who viewed Version *B*. Figure 11 presents this interaction graphically.

Table 7. Experiment 1 - Analysis 3: Results of the analysis of variance of the Group II (Y_3) - Group IV (Y_2) data

Source of Variation	SS	df[1]	MS	*F*
Ability level	162.241	1	162.241	2.785[b]
Program	558.036	1	558.036	9.579[a]
Ability level X Program	160.321	1	160.321	2.752[b]
Error	1339.906	23	58.257	
Total	2220.504	26		

[a] significant at the .01 level
[b] significant at the .25 level

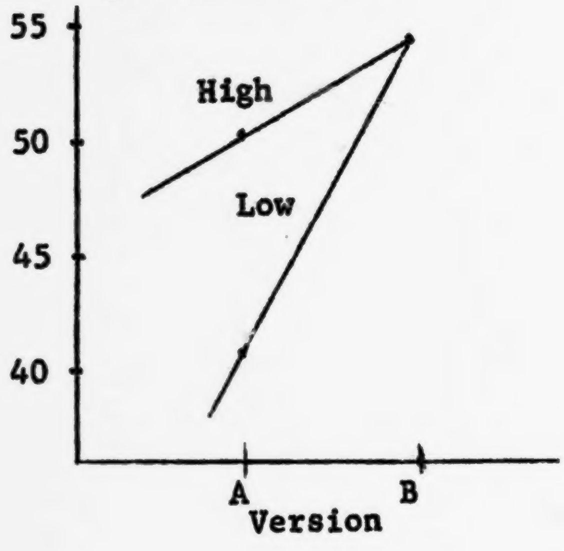
[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted when a cell entry was estimated by using the mean of the cell.

Table 8. Experiment 1 - Analysis 3: Cell means based on the pooling of normalized standard scores

		PROGRAM VERSION	
		A	B
ABILITY	HIGH	50.000 ↕	54.214 ↗
	LOW	↙ 40.471 ⟷	54.186

⟷ indicates significant difference between means

Figure 11. Graph of the Ability level X Program interaction



On the basis of these results, the following interpretations seem tenable for Experiment 1 - Analysis 3:

1. the high ability groups performed equally well on both versions (A and B);
2. when compared with low ability students who viewed Version A, low ability students who viewed Version B performed better;
3. Version A appeared better suited for the high ability students than for the low ability students; and
4. the revision procedures may have increased the difference between the programs, as evidenced in the significant Program effect.

Analysis 3A. The completeness of the data gathered in Experiment 1 allowed an extension of Analysis 3 to include the pooling of the Y_3 data from Group I and Group II and the Y_2 data from Group III, the retention group, and Group IV. The analysis of variance, summarized in Table 9, indicated 1) a significant interaction between Ability and Program, and 2) significant main effects for Ability level and Program. Table 10 and Figure 12 show the interaction cell means tabularly and graphically. Analysis of these means yielded differences, significant at the .05 level, 1) between the high and low ability groups who viewed Version A, 2) the high ability students who viewed Version B and the low ability students who viewed Version A, and 3) the low ability students who viewed Version A and the low ability students who viewed Version B. These results support the interpretation of the Analysis 3 results presented above.

Analysis 4. Presented in Table 11 are the results of the 2 x 2 x 2 analysis of variance of the Y_3 data. The following sources of variance were identified as significant: the Ability level X Program interaction, the Program X Group interaction, Group, and Program.

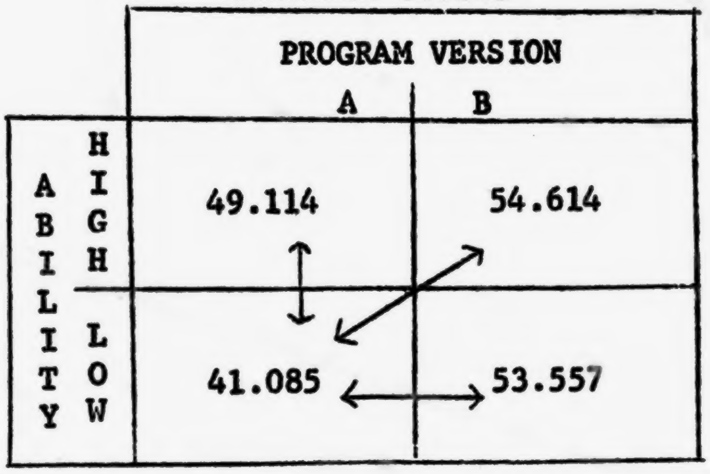
Table 9. Experiment 1 - Analysis 3A: Results of the analysis of variance of the $Y_3 - Y_2$ data from Administration 3.

Source of Variation	SS	df[1]	MS	F
Ability level	288.926	1	288.926	5.096[b]
Program	1130.403	1	1130.403	19.938[a]
Ability level X Program	170.103	1	170.103	3.000[c]
Error	2891.426	51	56.695	
Total	4480.858	54		

[a] significant at the .001 level
[b] significant at the .05 level
[c] significant at the .10 level

[1] In addition to the usual procedure of subtracting one df for the grand mean, one df was subtracted when a cell entry was estimated by using the mean of the cell.

Table 10. Experiment 1 - Analysis 3A: Cell means based on normalized standard scores



		PROGRAM VERSION	
		A	B
ABILITY	HIGH	49.114	54.614
	LOW	41.085	53.557

↔ indicates significant difference between means

Figure 12. Graph of the Ability level X Program interaction

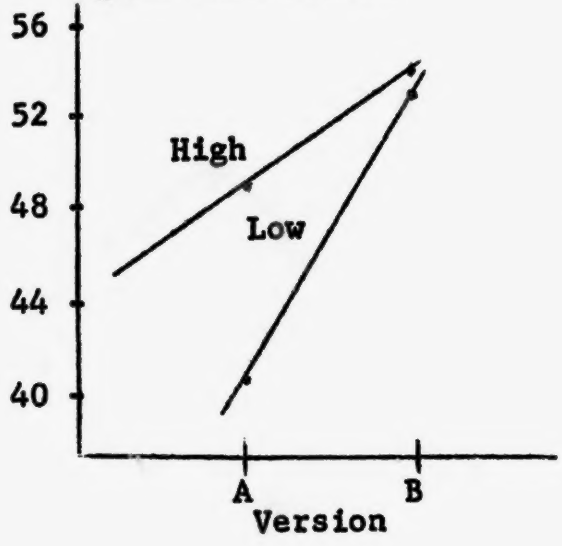


Table 11. Experiment 1 - Analysis 4: Results of the 2 x 2 x 2 analysis of variance of the Y_3 data from Group I and Group II

Source of Variation	SS	df[1]	MS	F
Ability level	12.500	1	12.500	2.426
Program	91.125	1	91.125	17.687[a]
Group	50.000	1	50.000	9.705[b]
Ability level X Program	12.500	1	12.500	2.426[d]
Ability level X Group	3.125	1	3.125	.606
Program X Group	32.000	1	32.000	6.211[c]
Ability level X Program X Group	3.125	1	3.125	.606
Error	118.500	23	5.152	
Total	322.875	30		

[a] significant at the .001 level
[b] significant at the .01 level
[c] significant at the .05 level
[d] significant at the .25 level

[1] In addition to the usual procedure of subtracting one df for the grand mean, one df was subtracted when a cell entry was estimated by using the mean of the cell.

The Newman-Keuls analysis of the Ability level X Program interaction cell means (Table 12) revealed differences, significant at the .05 level, 1) between the high and low ability students who viewed Version A, 2) between the high ability students who viewed Version A, and 3) between the low ability students who viewed Version A and the low ability students who viewed Version B. The graph of this interaction is presented in Figure 13.

The Newman-Keuls analysis of the Program X Group interaction cell means (Table 13) revealed differences, significant at the .05 level, 1) between Group I and Group II on Version A, 2) between the Group I students who viewed Version B and the Group II students who viewed Version A, and 3) between the Group II students who viewed Version A and the Group II students who viewed Version B. The graph of this interaction is presented in Figure 14. Consideration of this analysis in relation to the preceding analyses suggests that a greater amount of practice and exposure decreases the difference in difficulty level between the two versions. On the one hand, the Group I subjects, who had seen p' and p'' prior to viewing p''', performed equally well on both versions; on the other hand, the Group II subjects, who viewed p' and p''', performed relatively poorly on Version A, but at a level comparable to Group I on Version B. The significant main effect for the group factor was attributed to sampling error, since the groups were assigned randomly.

On the basis of these results, the following interpretations seem tenable for Experiment 1 - Analysis 4:

1. the high ability students performed equally well on both versions (A and B);
2. low ability students performed better on Version B, the program written for them, than a similar group performed on Version A;
3. the high ability students performed better than the low ability students on the version written for high ability students, *i.e.*, Version A; and

Table 12. Experiment 1 - Analysis 4: Cell means based on obtained scores --Ability X Program interaction

		PROGRAM VERSION	
		A	B
ABILITY	HIGH	6.75	8.889
	LOW	4.25	8.889

↔ indicates significant difference between means

Figure 13. Graph of the Ability X Program interaction

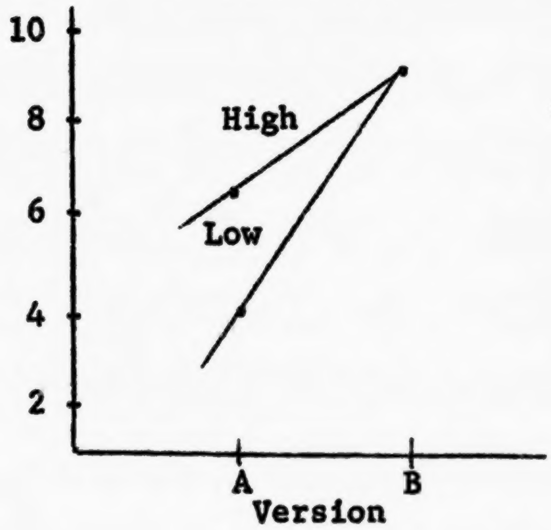
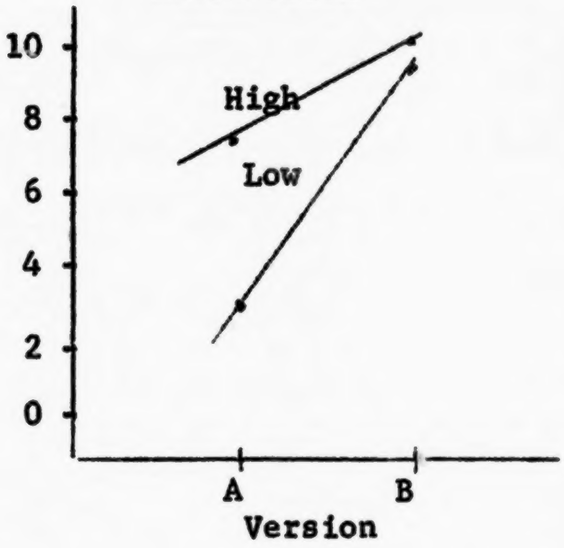


Table 13. Experiment 1 - Analysis 4: Cell means based on obtained scores --Program X Group interaction

		PROGRAM VERSION	
		A	B
GROUP	I	7.75	9.125
	II	3.25	8.625

↔ indicates significant difference between means

Figure 14. Graph of the Program X Group interaction



4. the difference in difficulty level between the two versions may be decreased by increasing the amount of practice and exposure.

Experiment 2

Analysis 1. The analysis of the pooled Y_1 data revealed an interaction between Ability and Program, as well as significant main effects for Ability and Program (.01 and .05 levels respectively). The reader is referred to Table 14 for a summary of the analysis of variance, to Table 15 for a presentation of the interaction cell means, and to Figure 15 for a graphical representation of the interaction. The Newman-Keuls analysis of the cell means showed differences, significant at the .05 level, 1) between the high and low ability students who viewed Version A, 2) between the high ability students who viewed Version B and the low ability students who viewed Version A, and 3) between the low ability students who viewed Version A and the low ability students who viewed Version B.

The results of Experiment 2 - Analysis 1 suggest that:

1. the high ability students performed equally well on both versions (A and B);
2. the low ability group performed better on the version written for them (Version B); and
3. the high ability students performed better than the low ability students on the version written for high ability students (Version A).

Analysis 2. The analysis of the pooled Y_2 and Y_1 data from Groups I and II yielded a significant main effect for Ability (.05 level), as shown in Table 16.

Analysis 3. When the Y_3 data for Group II and the Y_2 data for Group IV were pooled, a significant main effect was observed for the Program factor (.01 level), as indicated in Table 17. Such a difference, in general, indicates that one version, usually Version A, is more difficult than the other.

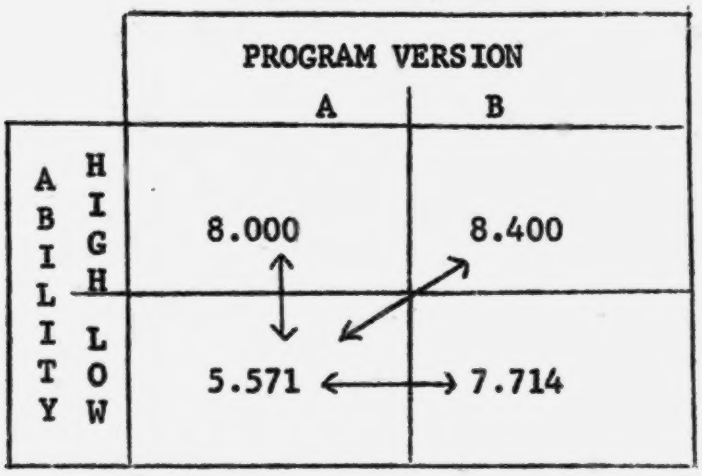
Table 14. Experiment 2 - Analysis 1: Results of the analysis of variance of the Y_1 data

Source of Variation	SS	df[1]	MS	*F*
Ability level	34.039	1	34.039	8.909[a]
Program	22.708	1	22.708	5.943[b]
Ability level X Program	10.579	1	10.579	2.769[c]
Error	187.218	49	3.821	
Total	254.544	52		

[a] significant at the .01 level
[b] significant at the .05 level
[c] significant at the .25 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted for each of three cell entries estimated by using the mean of the cell.

Table 15. Experiment 2 - Analysis 1: Cell means based on obtained scores



		PROGRAM VERSION	
		A	B
ABILITY	HIGH	8.000	8.400
	LOW	5.571	7.714

↔ indicates significant difference between means

Figure 15. Graph of the Ability level X Program interaction

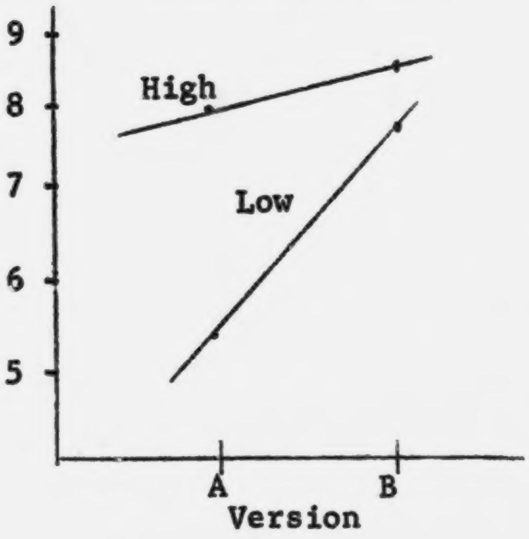


Table 16. Experiment 2 - Analysis 2: Results of the analysis of variance of the Group I (Y_2) - Group II (Y_1) data

Source of Variation	SS	df[1]	MS	*F*
Ability level	472.321	1	472.321	5.512[a]
Program	79.566	1	79.566	.929
Ability level X Program	19.892	1	19.892	.232
Error	1713.789	20	85.689	
Total	2285.568	23		

[a] significant at the .05 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted for each of four cell entries estimated by using the mean of the cell.

Table 17. Experiment 2 - Analysis 3: Results of the analysis of variance of the Group II (Y_3) - Group IV (Y_2) data

Source of Variation	SS	df[1]	MS	*F*
Ability level	2.813	1	2.813	.052
Program	597.325	1	597.325	11.144[a]
Ability level X Program	13.945	1	13.945	.260
Error	804.016	15	53.601	
Total	1418.099	18		

[a] significant at the .01 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted when a cell entry was estimated by using the mean of the cell.

Table 18. Experiment 2 - Analysis 4: Results of the 2 x 2 x 2 analysis of variance of the Y_3 data from Group I and Group II

Source of Variation	SS	df	MS	*F*
Ability level	13.140	1	13.140	3.737[a]
Program	6.890	1	6.890	1.960
Group	1.265	1	1.265	.359
Ability level X Program	2.642	1	2.642	.751
Ability level X Group	3.517	1	3.517	1.000
Program X Group	.392	1	.392	.111
Ability level X Program X Group	.138	1	.138	.039
Error	28.125	8	3.516	
Total	56.109	15		

[a] significant at the .10 level

__Analysis__ __4__. The results of the 2 x 2 x 2 analysis of variance of the Y_3 data appear in Table 18. Ability level was identified as a factor significant at the .10 level. This result tends to agree with the results of Analysis 2, for the same groups were involved in both analyses.

Experiment 3

Analysis 1. The analysis of variance of the Y_1 data revealed 1) an interaction between Ability and Program, and 2) a main effect for the Ability factor significant at the .10 level. These results appear in Table 19. When the Newman-Keuls technique was applied to the cell means (Table 20), differences significant at the .05 level were located 1) between the high and low ability students who viewed Version A, and 2) the high ability students who viewed Version B and the low ability students who viewed Version A. This interaction is graphed in Figure 16.

These results appear to be contingent upon the nature of the Original Program p'. When the topic, the graphing of inequalities, was selected for this experiment, it was presumed that the students would be familiar with linear graphing procedures. In actuality, the students' responses indicated that an intensive review of linear graphing was needed. Thus, the significant difference in ability level probably reflects the ability to handle novel situations or to recall some of the procedures involved in linear graphing.

Analysis 2. When the Y_2 data from Group I were pooled with the Y_1 data from Group II, the analysis of variance revealed a significant main effect (.05 level) for Ability, as shown in Table 21.

Analysis 3. Table 22 shows the results of the analysis of variance of the Y_3 and Y_2 data from Groups II and IV respectively. Ability level was identified as a main effect significant at the .001 level.

Table 19. Experiment 3 - Analysis 1: Results of the analysis of variance of the Y_1 data

Source of Variation	SS	df[1]	MS	F
Ability level	19.255	1	19.255	3.467[a]
Program	2.750	1	2.750	.496
Ability level X Program	11.158	1	11.158	2.014[b]
Error	271.453	49	5.540	
Total	304.616	52		

[a] significant at the .10 level
[b] significant at the .25 level

[1] In addition to the usual procedure of subtracting one df for the grand mean, one df was subtracted for each of three cell entries estimated by using the mean of the cell.

Table 20. Experiment 3 - Analysis 1: Cell means based on obtained scores

		PROGRAM VERSION	
		A	B
ABILITY	HIGH	3.500	3.988
	LOW	1.179	2.470

↔ indicates significant difference between means

Figure 16. Graph of the Ability level X Program interaction

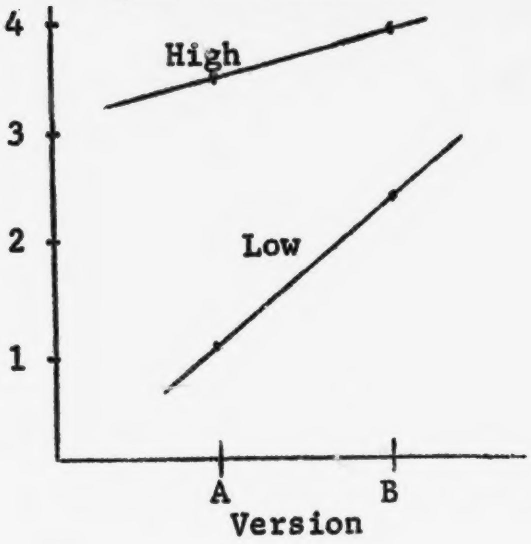


Table 21. Experiment 3 - Analysis 2: Results of the analysis of variance of the Group I (Y_2) - Group II (Y_1) data

Source of Variation	SS	df[1]	MS	*F*
Ability level	275.538	1	275.538	4.270[a]
Program	52.788	1	52.788	.818
Ability level X Program	.008	1	.008	.000
Error	1613.179	25	64.527	
Total	1941.513	28		

[a] significant at the .05 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted for each of three cell entries estimated by using the mean of the cell.

Table 22. Experiment 3 - Analysis 3: Results of the analysis of variance of the Group II (Y_3) - Group IV (Y_2) data

Source of Variation	SS	df[1]	MS	*F*
Ability level	1020.510	1	1020.510	19.924[a]
Program	136.804	1	136.804	2.671
Ability level X Program	.511	1	.511	.010
Error	973.165	19	51.219	
Total	2130.990	22		

[a] significant at the .001 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted when a cell entry was estimated by using the mean of the cell.

Analysis 4. The results of the 2 x 2 x 2 analysis of variance are presented in Table 23. The Program X Group interaction proved to be significant at the .10 level, and a significant main effect (.10 level) was observed for the Ability level factor. The Newman-Keuls analysis of the Program X Group interaction cell means (Table 24) revealed no difference significant at the .05 level.

Experiment 4

Analysis 1. The analysis of variance of the pooled Y_1 data indicated a significant main effect (.01 level) for the Ability factor, as shown in Table 25.

Analysis 2. Table 26 presents the results of the analysis of variance of the Y_2 data from Group I and the Y_1 data from Group II. A significant main effect for the Ability factor was observed at the .01 level. The cell means for the Ability level X Program interaction are presented in Table 27, and a graphical representation appears in Figure 17. Analysis of these means yielded differences, significant at the .05 level, 1) between the high and low ability groups who viewed Version A, 2) the high ability students who viewed Version B and the low ability students who viewed Version A, and 3) the low ability group which viewed Version A and the similar group which viewed Version B.

Although the data were somewhat sparse in this analysis, the following tentative interpretations may be noted for Experiment 4 - Analysis 2:

1. the high ability groups displayed an even performance level across versions;
2. the high ability students performed better than the low ability students on the version written for high ability students (Version A); and
3. Version A seemed better suited for the high ability students than for the low ability students.

Table 23. Experiment 3 - Analysis 4: Results of the 2 x 2 x 2 analysis of variance of the Y_3 data from Group I and Group II

Source of Variation	SS	df[1]	MS	*F*
Ability level	25.011	1	25.011	3.848[a]
Program	12.761	1	12.761	1.963
Group	14.261	1	14.261	2.194
Ability level X Program	.510	1	.510	.078
Ability level X Group	3.760	1	3.760	.578
Program X Group	21.093	1	21.093	3.245[a]
Ability level X Program X Group	4.594	1	4.594	.707
Error	97.500	15	6.500	
Total	179.490	22		

[a] significant at the .10 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted when a cell entry was estimated by using the mean of the cell.

Table 24. Experiment 3 - Analysis 4:
Cell means based on obtained scores--
Program X Group interaction

		PROGRAM VERSION	
		A	B
GROUP	I	7.083	6.667
	II	3.667	7.000

Table 25. Experiment 4 - Analysis 1: Results of the analysis of variance of the Y_1 data

Source of Variation	SS	df[1]	MS	F
Ability level	57.111	1	57.111	8.561[a]
Program	9.846	1	9.846	1.476
Ability level X Program	2.971	1	2.971	.445
Error	180.115	27	6.671	
Total	250.043	30		

[a] significant at the .01 level

[1] In addition to the usual procedure of subtracting one df for the grand mean, one df was subtracted when a cell entry was estimated by using the mean of the cell.

Table 26. Experiment 4 - Analysis 2: Results of the analysis of variance of the Group I (Y_2) - Group II (Y_1) data

Source of Variation	SS	df[1]	MS	F
Ability level	1219.922	1	1219.922	9.862[a]
Program	359.552	1	359.552	2.907
Ability level X Program	390.728	1	390.728	3.159[b]
Error	1855.428	15	123.695	
Total	3825.630	18		

[a] significant at the .01 level
[b] significant at the .10 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted when a cell entry was estimated by using the mean of the cell.

Table 27. Experiment 4 - Analysis 2: Cell means based on normalized standard scores

		PROGRAM REVISION	
		A	B
ABILITY	HIGH	54.440	54.080
	LOW	29.980	47.300

⟷ indicates significant difference between means

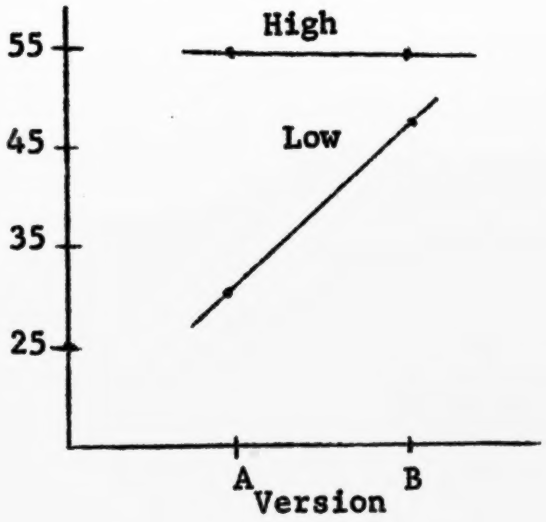


Figure 17. Graph of the Ability level X Program interaction

Discussion

Since statistically significant interactions resulted in several analyses in the different experiments, the investigators have concluded that the program treatment differentially affected the learning of students at different ability levels. The data supporting this conclusion are shown in the recurring pattern of learning outcomes and may be summarized as follows:

1. the high ability students performed equally well on Versions A and B,
2. in the case of Version A, the high ability students consistently performed at a higher level than did the low ability students, and
3. in most instances, the low ability students performed better on the program written for them, *i.e.*, Version B, than did the low ability students who viewed Version A.

The consistent performance of the high ability students is not surprising. Most placed in the upper quartile on the STEP. In addition, ceiling effects due to either the elementary nature of the introductory topics and/or maximum learning may have accounted for their performance. The difference in the performance of the high and low ability groups on Version A was anticipated, since pupils were divided with respect to the ability criterion. This division became evident in the experimentation when Ability level appeared repeatedly as a significant main effect.

The improved performance of the low ability group on Version B, together with the points just made, suggests the apparent need for at least two versions of programs in presenting many instructional topics. The significant main effect for the Program factor, which resulted in several analyses, shows that it is possible to write differing versions within one topical area such that the learning progress of students is differentially affected.

The specific differences of the two types of programs used in the Program treatments are summarized in Foldout A. The type of Program treatments hypothesized to efficiently and effectively facilitate the learning of students of relatively high ability differ significantly from the Program treatments hypothesized to efficiently and effectively facilitate the learning of students of relatively low ability. These empirically demonstrated differences are as follows:

1. The average length of time of the recorded scripts for Version _A_ was 4.3 minutes while the average time for Version _B_ was 9.4 minutes.
2. The average number of 2 x 2 slides used was 13 slides in Version _A_ and 30 slides in Version _B_.
3. The average number of response items in the student workbook for Version _A_ was 7 items, while for Version _B_ the average number was 18.
4. The average number of program stops needed for students to complete workbook exercises was 5 stops for Version _A_ and 16 stops for Version _B_.

These observations strongly suggest that any future applications of the programs developed in this project should involve careful consideration of the student group or groups who will receive the instruction.

FOLDOU

Summary of Program

CHARACTERISTIC	EXPERIMENT 1 Scientific Notation						EXPERIMENT 2 System Without Numbers						Grap	
	p'		p''		p'''		p'		p''		p'''		p'	
	A	B	A	B	A	B	A	B	A	B	A	B	A	B
I. Program Lengths and Stops:														
1. Minutes of script as tape recorded	1.5	7.1	1.8	8.9	1.8	9.7	6.5	10.5	5.9	11.7	6.1	11.7	5.4	8.7
2. Minutes for classroom operation[a]	4.2	16.8												
3. Number of workbook stops	2	14	2	15	2	17	6	15	4	15	3	15	10	18
II. Visual Displays:														
4. Number of 2 x 2 slides	6	30	6	23	6	26	11	28	20	37	15	37	19	32
5. 16 mm movie film							Three movie clips for each version and each administration of Experiment 2							
III. Student Learning Materials:														
6. Workbooks	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7. Number of response items	4	19	4	20	4	22	13	19	11	19	7	19	14	18
8. Number of items in post-test	10	10	10	10	10	10	10	10	13	13	13	13	9	9
9. Manipulatable materials							Cardboard rectangle and base diagram for each student for each version and administration in Experiment 2							
IV. Administrations:														
10. Number of times program was administered	2	2	2	2	1	1	2	2	2	2	1	1	2	2
11. Number of students participating in the administrations	28	28	19	19	15	16	28	25	16	13	11	9	28	26

[a] These time lengths include recorded script, automated display changes, and students' workbook responses. Only two observations of this total operational time were obtained due to practical difficulties. Rough estimates may be made by considering number of displays and number of response items.

[b] The p''' programs of Experiment 4 were not administered due to conflicts of activities in the school calendar and schedule.

Note : 1) p' designates the administration of the first version (i.e., original programs A and B) of the program; p'' designates administration of the second version (i.e., resulting from first revisions); and p''' designates administration of the third version (i.e., resulting from second revisions).

2) A designates Versions A, the programs prepared for learners of relatively high ability. B designates Versions B, the programs prepared for learners of relatively low ability.

OLDOUT A

rogram Characteristics

EXPERIMENT 3 Graphing of Inequalities						EXPERIMENT 4 Inequalities and the Number Plane						SUMMARY TOTALS			
p'		p''		p'''		p'		p''		p'''		Versions A: 12 Programs		Versions B: 12 Programs	
A	B	A	B	A	B	A	B	A	B	A	B	Sum	Average	Sum	Average
5.4	8.7	5.5	8.9	4.9	9.2	3.2	6.8	4.1	8.8	4.7	10.3	51.4	4.3	112.4	9.4
10	18	8	16	5	17	4	13	4	15	4	16	54	4.5	186	15.5
19	32	19	25	21	32	11	27	11	30	12	31	157	13.1	358	29.8
1	1	1	1	1	1	1	1	1	1	1	1	12	1	12	1
14	18	10	16	6	17	4	12	4	15	4	15	85	7.1	211	17.6
9	9	11	11	11	11	7	7	7	7	7	7	118	9.8	118	9.8
2	2	2	2	1	1	2	2	1	1	b	b	18	1.5	18	1.5
28	26	17	17	12	14	18	25	7	7	b	b	199	16.6	199	16.6

Chapter V

Outcomes: Discussion, Conclusions and Recommendations

The purpose of this chapter is to summarize the achievements of the project. These considerations involve a review of the general nature of the project intentions, the presentation of the conclusions arising from the developmental and experimental work, and the discussion of recommendations concerning the use and investigation of multimedia instructional resources in the high school classroom. It should be noted that the generality of these considerations is limited to projects similar to the one reported in this volume, which, of course, only deals with one particular approach to multimedia instruction and concerns only a small portion of the field of audio-visual techniques.

Project Intentions and Resulting Products

The initial conception and plan of the project was motivated by the desire to investigate the processes whereby a new automated multimedia teaching system could be effectively assimilated into the instructional repertoire of a rural high school. As observed in the foregoing pages, this objective involved three basic activities: a) the use of selected techniques of multimedia instruction, b) the application of selected principles of programed learning, and c) the instructional analysis and experimental investigation of selected topics of the mathematics curriculum

The conduct of these activities was intentionally undertaken within the restricted set of circumstances defined by a) involving only a single classroom group (numbering 60 to 65 students) for experimentation, b) using only units of learning material in mathematics which could be completed by all learners within a 25-minute period, and c) programing topics which appeared within the existing curriculum and instructional sequence. By performing research work within these limitations, it was hoped that the products and results of the research would have immediate utility for existing curriculum and instructional practices. In particular, it was hoped that the resulting programed multimedia units

1. would be immediately useable by the classroom teachers;
2. would facilitate student learning by increasing the individualization of instruction; and
3. would stimulate further curriculum developments and revisions.

The classroom evaluation of the developed programs was structured according to the requirements of an experimental design. The application of the design was important for two general reasons: a) it allowed the meaningful evaluation of the measurements of student learning, and b) it focused sharply on the need for precise preparation of programs, smooth coordination of materials and equipment for media presentation, and careful construction of learning materials used by the students.

Resulting from these several endeavors are three major products:

1. knowledge concerning the assimilation of new multimedia technology into the high school classroom and the effective use of this technology;
2. guidelines for the preparation of programed multimedia instructional units; and

3. eight tested multimedia programs ready for use in the high school classroom.

The knowledge acquired reveals the feasibility and effectiveness with which multimedia instruction can be developed within the context of existing classroom practices and curriculum patterns. The specific details of this finding are contained in the previous chapters and are summarized in the conclusions presented in the next section. The guidelines resulting from the research efforts are presented as the Checklist in Part II of this report. It is hoped that the Checklist will stimulate and facilitate the work of others who engage in the development of programed units. The eight programs produced are partially presented in Part III of this report. The final forms of these programs have been packaged independently of this report and include the fully programed tape recording, correctly ordered visuals ready for machine loading, the accompanying Student Workbook, and a copy of the complete Programing Layout and Worksheet for the scripts and visual displays.

Conclusions

Based upon the goals, methods, and experimental outcomes of the project, four conclusions have been reached:

1. The assimilation of new programed multimedia instruction procedures can be effectively achieved within the existing classroom and curriculum practices. By this means, instructional innovations evolve from known circumstances rather than from sources less intimately related to existing practices.
2. The development of new instructional programs should involve members of the teaching staff. Participation by the teaching staff is essential for insuring that the content of the programed units fits into the established curriculum sequence and that staff mem-

bers become skilled in the operations of the new techniques. Unless these conditions are fulfilled, continued effective use of the programed units is unlikely, and the flexibility afforded by the new procedures will be neglected.

3. The facilitation of learning by students of relatively high ability and students of relatively low ability is most effectively achieved by the construction of different types of programs. It has been demonstrated that the two types of programs used differ with respect to the amount of taped verbal instructions, the number of visuals used, and the number of workbook responses required of the learners. This conclusion implies that instruction of a heterogenous group of students by a single multimedia program is likely to be inefficient and ineffective in terms of some subgroup or subgroups of students.

4. The development of a multimedia program should proceed according to a systematic plan which includes opportunities for a) trial testing of the program with a few selected students, and b) program revision on the basis of the obtained student responses. The results of the experimental analyses evidence the effectiveness of the programs which undergo such revision. These developmental procedures increase in importance when the programs are being prepared by people inexperienced in program development.

Recommendations

As the project staff has reviewed the overall efforts and outcomes of the research, nine recommendations seem to be important for interpreting the meaning of the reported work in terms of future efforts of a similar nature. These nine recommendations concern four areas: the installation and arrangement of multimedia equipment in the classroom, the involvement of members of the school staff, the planning of program development, and the focus of further research.

Recommendations Concerning the Selection and Location of Multimedia Facilities

One of the first tasks undertaken was the construction of four very short programs designed to test the operational capabilities of the equipment which had been installed in the classroom prior to the proposal of the project. During these early tests various advantages and disadvantages of the facilities were highlighted. The location of the control console in the front of the classroom was found to be very convenient. This location allowed the instructor to coordinate the equipment automated by the control console with the non-programed equipment, such as chalkboards and overhead projector. However, it is desirable that the console be placed so that the instructor can stand at it without blocking the students' view of the screen. Also, it was difficult for the instructor to help the students individually, since his presence at the console controls was necessary. The addition of a remote stop-start switch on the end of a 60-foot cord allowed the instructor freedom to move among the students while simultaneously retaining control of the rate of program presentation.

The large rear projection screen, shown in Photograph 1 (page viii), was most advantageous. One drawback, however, was the location of the overhead projector screen which overlapped the rear projection screen. As this mounting was never corrected, the use of the overhead projector was not included in the programing for the four experiments.

The automated equipment which could be controlled by the console was initially limited, for the console did not allow more than one projector to project a display at any given time. This feature was modified electronically to allow the operational coordination of two

2 x 2 slide projectors. The lenses of the 2 x 2 projectors were also limited in size of the projected image; this limitation was corrected by the purchase of special short-throw lenses.

Other modifications of the equipment which were noted as desirable but not undertaken were the following: an easier method of calibration for the console meters which indexed the percentage of students responding to each possible response alternative given in the workbook; a device for automatically storing student scores; a mechanism for reversing the movement of slide and strip film machines at the control console; and a remote control focus switch for the projectors (see Appendix F).

Based on the above observations, the following recommendations are made:

> Recommendation 1: Plans for the physical layout of the room in which multimedia equipment will be used should be finalized only after careful consideration of the various types of circumstances in which the equipment will be used; specific points to be considered include console location, student seating arrangements, housing of projection equipment, and equipment accessories.
>
> Recommendation 2: Multimedia equipment should be purchased after careful consideration of the extent of flexibility needed for presentations and the feasibility of later converting or modifying the equipment to be installed.

Recommendations Concerning School Staff Skill and Involvement in Program Development and Operation

One of the major reasons for initiating this project was the desire to promote and insure maximal use of the multimedia equipment. Although various demonstrations and in-service training sessions had been held, few staff members were skilled in the practice of program operations and development. These circumstances were partially occasioned by the limitations of the equipment, as described in the previous section, but were also caused by lack of sufficient experience in the actual process of program writing and production. Such expertise can be gained only by systematic trial and error efforts. The assistance offered by manufacturers is usually insufficient for staff members to gain the needed competencies, for they have limited time and energy to devote to the activities leading to familiarity and skill. Also, teachers are often likely to react to the flexible capabilities of modern instructional technology by equating diversity of operations with complexity and, thus, come to consider the machinery quite forbidding. Failure to fully acquaint staff with the procedures of multimedia programing will place these resources in the role of school trophies: a source of pride in their presence, but of little functional value.

It is important, therefore, to initiate programing projects in cooperation with staff members who can be intimately involved and can be given released time from classroom responsibilities. It should also be recognized clearly that the primary function of a teacher is teaching; from this viewpoint, the teacher is most qualified to select topics for program development. The involvement of staff should be augmented by needed expert resources from outside the school staff, especially when it is hoped that relatively long programs, or a series of programs, will be developed.

The following recommendations should be considered within the perspective of the guidelines for program development presented in Part II of this report:

> Recommendation 3: Provision should be made to instruct the staff in the use of the multimedia equipment and to

provide opportunities for them to prepare trial programs with which they can test their own skill and knowledge in selecting suitable content topics for program development.

Recommendation 4: Program development should be conducted by staff members, together with any needed additional resource people, according to a systematic plan and schedule of activities.

Recommendations Concerning the Development of Multimedia Programs

A major justification for the initiation of this project was the lack of available programs and the lack of detailed guidelines which could satisfactorily direct the development of multimedia programs appropriate for efficient and effective operation in the classroom. Too often discussion and experience concerning program development has been restricted to those persons already experienced in this method of instruction. It was hoped that one of the results of the project work would be the development of the needed guidelines based upon successful experiences of the project. Detailed guidelines are presented in Part II, and the related programs are illustrated in Part III. Complementary to the specific details are several general points involved in program development.

The first point is that the result of student participation in a multimedia instructional program should be that the student, upon completing the program, can demonstrate that he has learned something which he did not know before participating. For example, he should be able to write a number in scientific notation, which previously he was unable to do. Thus, the programer should have knowledge of the student's previous level of achievement and should be acquainted with instructional sequences which have allowed the student to successfully learn. This means that the classroom teacher plays a prominent role in program development, for he is the person who has knowledge of his students' past achievements and is familiar with instructional procedures which successfully facilitated student learning. The background knowledge and experiences of the teacher are vital to the design of the overall program planning, as well as to specific programing tasks, e.g., the construction of the student workbook.

Special note should be made of the role of the student workbook in the reported experimentation. This workbook should not be considered a test of learning; rather, it is a means of facilitating learning and a basis for revising and improving the multimedia programs. Response errors in the workbook should be considered indications of difficult or ineffective program sequences. Modifications and additions to the program should reduce the frequency of the response errors; that is, the detail of verbal instructions programed may be increased, and/or visual displays may be revised to facilitate the learner's insight and understanding.

One final point concerning program development is that such activities should be regularly discussed not only with the immediately involved members of the school staff, but also with other staff personnel. It will help the general morale of the school if all members are at least familiar with the goals and procedures of the project, and are occasionally provided with opportunities for viewing demonstration programs so that they might realistically consider undertaking their own program development. All members of the faculty should, of course, be notified of slight changes in schedules or student involvement occasioned by the developmental activities.

Recommendation 5: Since the teacher is in the best position to determine what responses will be new for the students, he a) should be intimately involved in program development, b) should be considered the primary source of information concerning the previous work of the students, and c) should assist with the administration of a diagnostic pretest designed to assess the pre-program achievement level of the students.

Recommendation 6: The student workbook should be considered an essential part of the learning process for students. The workbook responses should be used as the basis for program modifications.

Recommendation 7: A detailed plan of the steps involved in program development should be established for use in monitoring and evaluating progress of the project. Staff members not directly related to the project should be informed concerning the goals and procedures of the project, provided with opportunities to view demonstration programs, and notified of any activities which may affect their schedules.

Recommendations Concerning the Implications of the Project and the Focus of Further Research

A major factor investigated by the experimentation reported was the variation of learning abilities. This variation may be observed in any classroom group of students. It was hypothesized that students of differing mathematics ability would profit from instructional programs appropriate to their relative ability level. Thus, it was hoped that the programs would contribute towards the individualization of the curriculum and classroom instruction. The hypothesis was supported by the data reported in Chapter 4. The general implication of this finding is that if a multimedia program is not molded to fit the learning needs and abilities of a specific group of students, then the instructional procedures will decrease correspondingly in terms of their efficiency and effectiveness. This general implication is presented in Diagram A, which summarizes the experimental results in terms of

Diagram A

Efficiency and Effectiveness of Learning as Related to Student Group Characteristics and Program Version

Characteristics of Student Group	Program Version	
	Version A Designed for Learners of Relatively High Ability	Version B Designed for Learners of Relatively Low Ability
Group is composed of students with a heterogeneous range of learning abilities in the subject matter programed.	High ability students will learn efficiently and effectively. Low ability students will learn inefficiently and ineffectively.	High ability students will learn inefficiently but effectively. Low ability students will learn efficiently and effectively.
Group is composed of students with a homogeneous range of learning abilities in the subject matter programed.	High ability students will learn efficiently and effectively. Low ability students will learn inefficiently and ineffectively.	High ability students will learn inefficiently but effectively. Low ability students will learn efficiently and effectively.

efficiency and effectiveness of learning as related to student group characteristics and program version.

Although variation of student learning abilities is inherent in every group of students, the specific applications of these findings is conditioned by the characteristics of the students who participated in the reported experiments. The details concerning these characteristics are presented in Appendix E. The following comments summarize their characteristics. Sixty-two freshmen, the subjects of the first two experiments, obtained scores ranging from the eleventh percentile through the ninety-fifth percentile on the Mathematics Section of the STEP. Their median percentile score, used to divide the class into low and high ability groups, was 67; thus, scores in the low ability group ranged from the eleventh percentile to the sixty-seventh percentile, and the scores for the high ability group ranged from the sixty-eighth percentile through the ninety-fifth percentile. These learning abilities of students implied by these ranges influenced the process of program construction presented in Chapter II. Furthermore, the teacher of this class would have justification for presenting Version A to the high ability group and Version B to the low ability group. If it seemed impracticable to divide the group into ability levels for program presentation, the most appropriate compromise would be Version B; for both groups did equally well on that version, although the high ability group learned with relative inefficiency. Thus, in aligning students with programs, the classroom teacher must consider the range of ability levels and past classroom performance. Furthermore, he would have to weigh additional criteria in placing those students who obtained scores within a few points of the median and who may profit from assignment to the other group.

Consideration of pupil and program characteristics will provoke many researchable hypotheses. Experimentation under classroom conditions, of course, would include determination of the range of abilities so that appropriate topics are chosen and group assignments are made judiciously. The application of the experimental approach conducted in this project, however, would involve the same developmental and operational procedures even though the range of abilities might be smaller.

> Recommendation 8: The classroom teacher should consider the specific ability characteristics of a particular group of students before assigning students to an appropriate program and before developing a particular program.
>
> Recommendation 9: Individualization of a subject matter curriculum can be achieved progressively by developing and testing multimedia programs designed to fit the learning needs and abilities of a defined group of students. The factors studied in the reported experimentation should be investigated in the context of other subject matter areas. Additional factors which are amenable to further research include a) differential rates of information presented in multimedia programing as related to different levels of student learning ability; b) the length of response time needed by students of differing learning ability levels; c) the effects on learning associated with various types of media presentations and learning materials manipulated by the students during programed multimedia learning units; and d) the investigation of types of program versions as related to characteristics of groups and subgroups of students.

Chapter VI

Summary

The process of assimilating new knowledge of learning processes and recent technological advances into the existing instructional repertoire of schools presents a major problem to educational institutions when the results of previous experiences are not available. The present project focused on this problem by investigating the most effective ways in which programed mathematics instruction might be integrated into the new multimedia instructional technology.

Program development, initiated by the Mathematics Team Leader, was begun with a careful survey of the existing classroom procedures and instructional sequence. Included in the developmental procedures were the following major areas:

1. eight topics considered appropriate for multimedia presentation were selected from those topics in the existing instructional sequence;
2. normal classroom presentations of these topics were outlined;
3. each topic was outlined in the form of programed frames, and ideas for visuals which would facilitate instruction were suggested; and
4. two pilot programs were conducted to assess the work to date and to develop procedures for revisions, *i.e.*, program refinement.

To provide meaningful information concerning the program development, experimentation was begun and focused on the hypothesis that

> using a 2 x 2 factorial design in which one of the factors is Ability (High and Low) and the other factor is the Program (Version *A* and Version *B*), a statistical interaction will be observed between Ability and Program.

This hypothesis was tested in four studies, each involving an introduction to a mathematical topic; the four topics chosen were a) scientific notation, b) a system without numbers, c) the graphing of inequalities, and d) inequalities and the number plane.

The procedures for each study (topic) were the following:

1. two versions (A and B) of each program were prepared -- A for high ability students, and B for low ability students;
2. each version was revised twice, and repeated administrations of the original program and revisions were conducted within the framework of the experimental design;
3. the subjects, high school students, were divided into high and low ability groups on the basis of their performance on the Mathematics Section of the STEP; then, they were assigned at random to administration groups; and
4. measures of learning, devised to accompany each version, were administered to the students after they had viewed a particular program version.

Several analyses of variance, based on data resulting from the pooling of groups at various administration levels, were used to test the Ability by Program interaction hypothesis. Pertinent interactions were analyzed further by the Newman-Keuls technique for significant mean differences.

Several significant main effects and interactions were observed. Generally, the interactions assumed the following pattern:

1. the high ability students performed equally well on Versions A and B;
2. on Version A, the high ability students consistently performed better than the low ability students; and
3. the low ability students performed better on the program written for them, *i.e.*, Version B, than did the low ability students who viewed Version A.

On the basis of the experimental results and program development demonstration, the following conclusions were considered tenable:

1. The assimilation of new programed multimedia instructional procedures can be effectively achieved within the existing classroom and curriculum practices.
2. Members of the teaching staff should be involved in the development and operations of new instructional programs to insure continued effective use of the programed units, as well as flexibility of new procedures within the curriculum.
3. Learning is facilitated effectively among students of differing ability levels by the construction of programs commensurate with ability levels.
4. The development of a multimedia program should proceed according to a systematic plan which includes opportunities for a) trial testing of the program with a few selected students, and b) program revision on the basis of the obtained student responses.

The following recommendations were made to those public school personnel who have made the decision to assimilate multimedia equipment into the instructional repertoire of their school programs:

1. Plans for the physical layout of the room in which multimedia equipment will be used should be finalized only after careful consideration of the various types of circumstances in which the equipment will be used; specific points to be considered include console location, student seating arrangements, housing of projection equipment, and equipment accessories.
2. Multimedia equipment should be purchased after careful consideration of the extent of flexibility needed for presentations and the feasibility of later converting or modifying the equipment to be installed.
3. Provision should be made to instruct the staff in the use of the multimedia equipment and to provide opportunities for them to prepare trial programs with which they can test their own skill and knowledge in selecting suitable content topics for program development.

4. Program development should be conducted by staff members, together with any needed additional resource people, according to a systematic plan and schedule of activities.

5. Since the teacher is in the best position to determine what responses will be new for the students, he a) should be intimately involved in program development, b) should be considered the primary source of information concerning the previous work of the students, and c) should assist with the administration of a diagnostic pretest designed to assess the preprogram achievement level of the students.

6. The student workbook should be considered an essential part of the learning process for students; the workbook responses should be used as the basis for program modifications.

7. A detailed plan of the steps involved in program development should be established for use in monitoring and evaluating progress of the project. Staff members not directly related to the project should be informed concerning the goals and procedures of the project, provided with opportunities to view demonstration programs, and notified of any activities which may affect their schedules.

8. The classroom teacher should consider the specific ability characteristics of a particular group of students before assigning students to an appropriate program and before developing a particular program.

9. Individualization of a subject matter curriculum can be achieved progressively by developing and testing multimedia programs designed to fit the learning needs and abilities of a defined group of students. The factors studied in the reported experimentation should be investigated in the context of other subject matter areas. Additional factors which are amenable to further research include a) differential rates of information presented in multimedia programing as related to different levels of student learning ability; b) the length of response time needed by students of differing learning ability levels; c) the effects on learning associated with various types of media presentations and learning materials manipulated by the students during programed multimedia learning units; and d) the investigation of types of program versions as related to characteristics of groups and subgroups of students.

This report contains detailed discussion of these topics, lists recommendations for persons engaging in similar demonstrations and experimentation endeavors, and presents twelve instructional programs actually developed and used in the project.

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PART II

THE DEVELOPMENT OF PROGRAMED MULTIMEDIA INSTRUCTIONAL UNITS: A CHECKLIST AND AN EXAMPLE

THE DEVELOPMENT OF PROGRAMED MULTIMEDIA INSTRUCTIONAL UNITS: A CHECKLIST AND AN EXAMPLE

Introduction

The material contained in this part of the report is designed to consolidate the knowledge and experience gained during the project with respect to the production of programed multimedia instructional units. Part I reports the plans, experiments, and outcomes involved in assimilating new technolgoy into a school's instructional repertoire, as evidenced by the instructional effectiveness of the programs presented in Part III. The checklist and the example of work presented here summarize the specific steps taken to write a multimedia instructional program. The presentation of this checklist has been organized as a practical guide for those persons inexperienced in programing and program production.

The systematic arrangement and specification of these production guidelines provides a framework for efficient and effective program development. Production of multimedia learning units can be a rich and rewarding experience. New insights are gained concerning student learning behaviors and the consequences of certain teaching procedures. During the progress of such a project difficulties and frustrations are often encountered. It is hoped that the checklist and its related example will help others avoid errors and unnecessary waste of time, effort and money.

Underlying the developmental guidelines which follow is a particular conceptualization of the teaching-learning process. Of greatest importance is the view that effective learning results from clear and precise perceptual discriminations of the learner. For these discriminations to occur, learning behaviors need to be directed and controlled in terms of specific instructional objectives. Student involvement -- attention, interest, and concentration needs to be facilitated by psychologically sequenced experiences based on logically arranged, relevant subject matter. Correct behavioral responses need confirmation and reinforcement. Revision and improvement of instruction need to be based on behavioral feedback and related student reactions to the learning experience.

This conceptualization accounts for the particular approach used in the reported research. The guidelines presented are limited by the particular strategies employed in the project. These strategies focused on short units of learning in mathematics varying in time for 4 - 20 minutes. Although these strategies concern only a small portion of programed learning and multimedia instruction, it is believed that the guidelines have implications for a wider area of classroom instruction and school curriculum. The efficacy of this view, however, can only be supported by useful application of the reported knowledge and experiences.

Guidelines may be used for:

1. assessing the extent and utility of immediately available resources and the specification of additional resources necessary for successful program production;
2. determining the capabilities of interested persons, their willingness and their assignment to particular project duties and responsibilities;

3. testing staff motivation and involvement in new procedures of classroom teaching and learning;
4. scheduling and monitoring the organization and coordination of developmental work; and
5. defining the functions of outside resource personnel and sharply focusing their involvement and contributions within the total structure of project activities.

All the material presented is neither completely necessary nor completely sufficient. Its function is to stimulate and facilitate the production of efficient and effective learning experiences for all students, for those of relatively low ability as well as those of relatively high ability.

PART II - SECTION A

CHECKLIST for PROGRAM DEVELOPMENT

A Guide for Planning and Monitoring the Development of a Programed Multimedia Instructional Unit

CHECKLIST for PROGRAM DEVELOPMENT

Explanatory Note

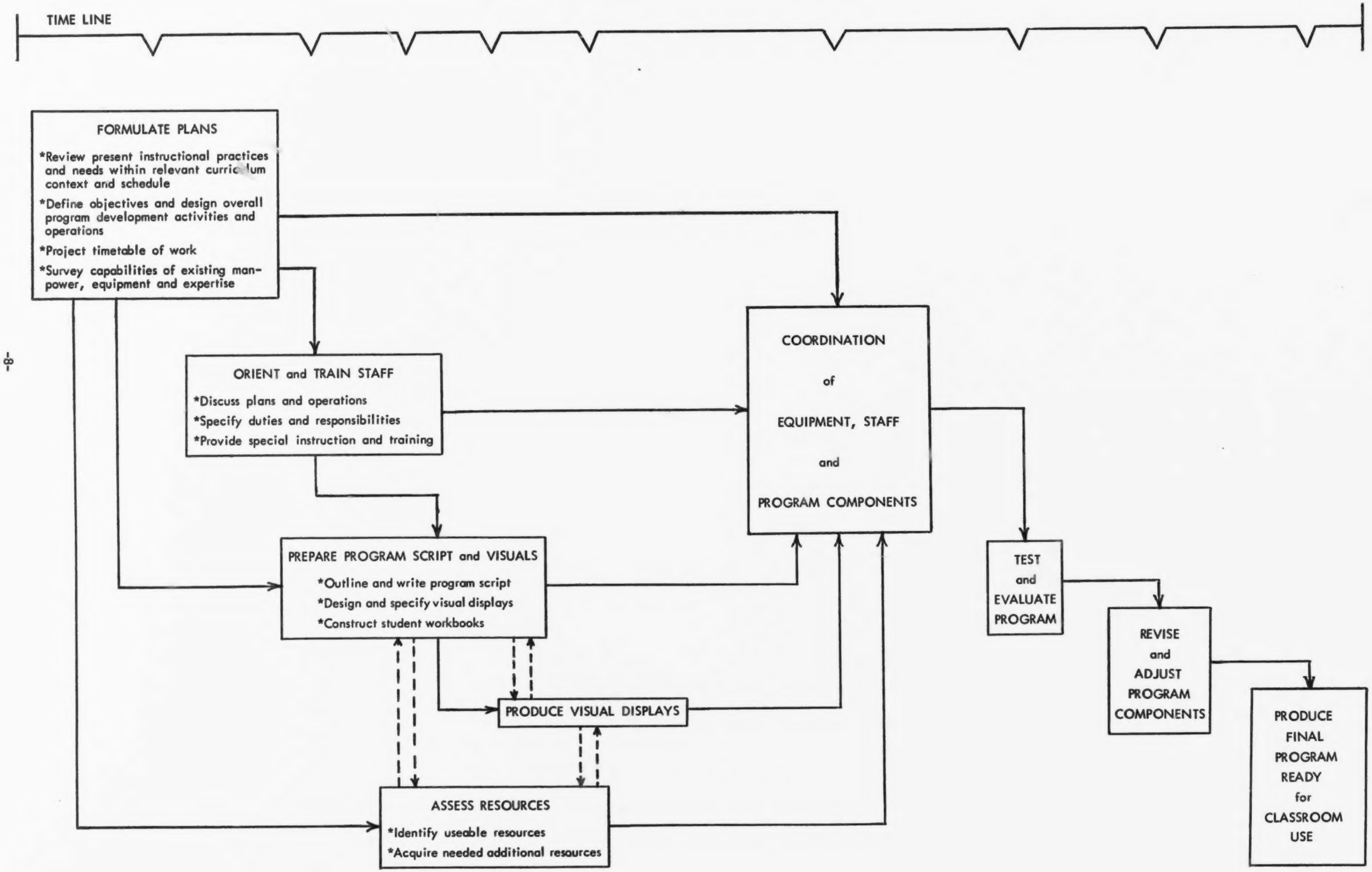
This checklist consists of statements defining tasks necessary for the development of a programed multimedia instructional unit. The purpose of this listing is to provide a tool useful for

a.) planning the development of a program, and

b.) monitoring the progress of programing work.

The user of the Checklist needs to develop his own particular method of using this tool. The most effective application of the listing by the inexperienced programmer will result from gaining a general understanding of the total process of program production. This understanding might be obtained by: 1) considering the particular tasks listed in the context of the total program demands, 2) reviewing the flow of work and operations outlined in the Flowchart of Program Development Activities of this Guide, 3) studying the program writing process exemplified in Section B of the Guide, and 4) participating, when possible, as a learner in the administration of the type of multimedia instructional program described in Parts I and III of this report. It is hoped the user will avoid rigid application of the Checklist. Its use should facilitate management and assessment of the programing project and should pinpoint successful progress.

FLOWCHART OF PROGRAM DEVELOPMENT ACTIVITIES


TIME LINE
FORMULATE PLANS
*Review present instructional practices and needs within relevant curriculum context and schedule
*Define objectives and design overall program development activities and operations
*Project timetable of work
*Survey capabilities of existing man-power, equipment and expertise
ORIENT and TRAIN STAFF
*Discuss plans and operations
*Specify duties and responsibilities
*Provide special instruction and training
PREPARE PROGRAM SCRIPT and VISUALS
*Outline and write program script
*Design and specify visual displays
*Construct student workbooks
PRODUCE VISUAL DISPLAYS
ASSESS RESOURCES
*Identify useable resources
*Acquire needed additional resources
COORDINATION of EQUIPMENT, STAFF and PROGRAM COMPONENTS
TEST and EVALUATE PROGRAM
REVISE and ADJUST PROGRAM COMPONENTS
PRODUCE FINAL PROGRAM READY for CLASSROOM USE

CHECKLIST for PROGRAM DEVELOPMENT

A Guide for Planning and Monitoring the Development of A Programed Multimedia Instructional Unit

Tasks — Check When Task Completed

I. PLANNING THE DEVELOPMENT OF THE PROGRAM

A. Assess the specific needs of classroom instruction which will be fulfilled by development of the program:

1. Review, within the perspective of the subject matter curriculum, the particular content area which is being considered for multimedia programing. ____________

2. Write a brief statement of reasons for considering the proposed content area for programing. ____________

3. List the major topics relevant to the content area which will be programed. ____________

4. Other (Specify) ______________________ ____________

B. Survey the entire set of tasks and jobs to be completed to obtain a gestalt of program development and project planning (i.e., review all items of this Checklist and specify additions where necessary). ____________

C. Identify the specific resources and equipment which are available or can be made available. ____________

D. Project the time schedule of the program development by estimating the completion date for each major group of tasks. ____________

E. Other (Specify) ______________________ ____________

Checklist for Program Development -- continued

Tasks	Check When Task Completed
II. PREPARATION OF THE PROGRAM SCRIPT	
A. Select one major topic to be programed.	__________
B. Write down each step of the classroom procedure which is presently used for teaching the selected content learning unit.	__________
C. Identify the content objective(s) of the selected topic.	__________
D. Make a judgment, using objective evidence if possible, concerning the level of achievement of the total group and its subgroups for the selected learning unit.	__________
E. Decide, on the basis of student learning achievement, which type of program will be prepared:	
1. Classify the group and/or its subgroups as to the degree of variability of learning ability.	__________
2. Decide whether the program will be prepared for the group as a whole or for a particular subgroup or subgroups.	__________
N.B.: Research evidence strongly suggests that only in rare cases when a total group is very homogeneous will there be justification for preparing one program for total group instruction. Thus a second (or additional) program may be developed simultaneously or scheduled for later development.	
3. Other (Specify) __________	__________
F. Partition the outlined learning unit into its parts so that a part defines a set of learning items which can be completed in one unit of daily instruction.	__________

Checklist for Program Development -- continued

Tasks | Check When Task Completed

G. Select the particular part of the partitioned topic (i.e., the content learning unit) which is to be programed. ____________

H. Write the Basic Program Script:

  1. Draft the script for the selected unit or units of daily instruction by using the description of the presently used classroom procedures and the list of instructional objectives:

    a. Specify in a logical sequence the behavior(s) which are associated with the objective(s) and which would verify that the objective(s) had been attained, i.e., terminal learning behavior(s). ____________

    b. For each of the specified behaviors write one or more subject matter examples of the behavior together with an instructional comment on the examples. ____________

    c. Recheck comments and examples for logical order. ____________

    d. Other (Specify) ____________________ ____________

I. Complete the Program Script:

  1. Prepare an introduction to the Basic Program Script to motivate learning by writing an introductory script and sketching needed visual displays to stimulate the learner's interest. ____________

  2. Prepare a conclusion to the Basic Program Script by writing a concluding script and sketching needed visual displays. ____________

  3. Review the Basic Program Script so as to identify and sketch additional visual displays needed for pin-pointing attention, facilitating discrimination and maintaining motivation and concentration. ____________

Checklist for Program Development -- continued

Tasks	Check When Task Completed
4. Specify the procedures by which students will respond to exercises and problems given in the Basic Program Script: follow each of the specified response procedures with feed-back which will provide the learner with a model of the correct response.	______
5. Assemble in proper sequence the introduction, Basic Program Script and conclusion thereby compiling the Program Script and Visuals.	______
6. Edit the Program Script and Visuals for continuity, variety and relevance.	______
7. Other (Specify) ______________________	______
III. PRODUCTION OF THE VISUAL MATERIALS	
A. Design the proposed visuals:	
1. Check the sketches for continuity (*e.g.*, consistency of notation throughout).	______
2. Decide, within the limits of available resources and equipment, the most effective method for displaying the visualized fact, concept, behavior and/or elicited response.	______
3. Prepare detailed plans of each visual:	
a. State the specifications for each visual, including mode of production, using a standard slide specification form.	______
b. Number each specified display according to its place in the sequence of the Program.	______
c. Other (Specify) ______________________	______
4. Other (Specify) ______________________	______

Checklist for Program Development -- continued

Tasks	Check When Task Completed
B. Initiate arrangements and procedures for production of the visuals:	
1. Discuss the sketches and specifications with the person(s) who will produce the visuals.	________
2. Schedule the production of the visuals.	________
3. Request preparation of a number of opaque (blank) slides.	________
4. Other (Specify) ________________	________
C. Review the produced visuals:	
1. Review the produced visuals using the classroom multimedia equipment; in particular check the visuals for consistency with the Program Script and for their readability from all points in the room -- note any needed changes or modifications.	________
2. Communicate the changes and additions to the production person(s) using the standard production form.	________
3. Check the final set of produced visuals following the procedures used before.	________
4. Store the visuals in the most useful and operational manner.	________
5. Other (Specify) ________________	________
IV. COORDINATION OF THE PROGRAM SCRIPT AND THE VISUAL DISPLAYS	
A. Prepare a detailed Program Worksheet so as to show the coordination of script and visuals:	

Checklist for Program Development -- continued

Tasks	Check When Task Completed
1. Transfer the Program Script and Visuals to the Program Worksheet:	
a. Designate on the Program Worksheet the media mode corresponding to each screen or part of the screen.	________
b. Type the Script onto the Worksheet.	________
c. Sketch and number each visual display in the appropriate column on the Worksheet.	________
d. Other (Specify) ________________	________
2. Review the sequence of displays in order to identify the points at which a projection screen should be blank and place a blank (opaque) slide wherever needed in the sequence of visual displays.	________
3. Check the Worksheet sequence for correctness of the script-visuals correspondence.	________
4. Mark and number at the exact points in the script all program cues needed for automating the displays, both visual projections and blanks, stop-start points, and any other activities.	________
5. Other (Specify) ________________	________
V. PREPARATION OF THE PROGRAMED TAPE RECORDING AND THE EQUIPMENT	
A. Record the Program Script:	
1. Attach an identification label to the tape reel.	________
2. Record at the beginning of the tape the identification data, i.e., the title of the program, date and name of programer(s).	________

Checklist for Program Development -- continued

Tasks	Check When Task Completed
3. Record the Program for actual presentation by completing the following tasks simultaneously:	
a. Record the script, allowing pauses for sound cues and stop pauses.	____________
b. Record a sound cue at each point in the script where the students are to make a response.	____________
c. Record a silence (e.g., five seconds) following each control cue indexing a student response to allow the instructional administrator sufficient time to activate the remote stop/start button.	____________
d. Other (Specify) ______________________	____________
B. Record the equipment control cues:	
1. Learn the operations of the console control system by studying the equipment instruction manual.	____________
2. Record a control cue for each specified cue point in the script sequence.	____________
3. Record control cues for any additional reasons.	____________
4. Other (Specify) ______________________	____________
C. Arrange the materials and equipment for a full-scale test of the prepared program:	
1. Load each media machine with the ordered visual displays.	____________
2. Place a Program Worksheet at the control console.	____________

Checklist for Program Development -- continued

Tasks	Check When Task Completed
3. Position the equipment and set all controls of the equipment:	
a. Focus the projectors.	______
b. Operationally test each machine via the control console and the remote stop/start switch.	______
c. Check the setting of volume and tone control.	______
d. Check availability of spare projector bulbs.	______
4. Other (Specify) ______________________	______
D. Arrange the classroom environment:	
1. Set the light level.	______
2. Position student response buttons.	______
3. Position and/or adjust the projection screens.	______
4. Other (Specify) ______________________	______
E. Other (Specify) ______________________	______
VI. CONSTRUCTION OF STUDENT WORKBOOKS	
A. Review the Program Worksheet to identify the responses expected of the student:	
1. Identify all computational exercises.	______
2. Identify all verbally presented problems.	______
3. Identify all manipulations of the materials.	______
4. Other (Specify) ______________________	______

Checklist for Program Development -- continued

Tasks	Check When Task Completed
B. List the identified exercises, problems and manipulations in the sequence that students will perform them during the program.	______
C. Prepare the Workbook by writing down in order the directions for each exercise, problem and/or manipulation.	______
D. Review the list of Worksheet items for consistency of notation and explanation.	______
E. Number the exercises according to the numbering system used on the tape recording and on the visuals.	______
F. Write a posttest.	______
G. Duplicate sufficient Workbooks for trial purposes.	______
H. Other (Specify) ______________________	______
VII. TEST AND EVALUATE THE PROGRAM	
A. Prepare all equipment, materials and classroom for a trial run of the program.	______
B. Start the Program and follow it through using the Program Worksheet:	
1. Note any discrepancies.	______
2. Note any needed additions or revisions.	______
3. Other (Specify) ______________________	______

Checklist for Program Development -- continued

Tasks | Check When Task Completed

C. Make any needed additions, revisions and/or changes. ______

D. Instruct two (or three) students by the use of the Program:

   1. Describe the general purpose of the session to the students. ______

   2. Note beginning and ending time. ______

   3. Other (Specify) ______________ ______

E. Discuss the experimental session with the students:

   1. Solicit their comments on the procedure and operation. ______

   2. Obtain their reactions to the Workbook. ______

   3. Other (Specify) ______________ ______

F. Score the Workbooks and posttests of the students. ______

G. Review the Program Worksheet and Student Workbook in terms of the students' performance and scores and note any needed changes. ______

H. Complete any needed changes and/or additions. ______

I. Other (Specify) ______________ ______

VIII. PREPARATION FOR FULL-SCALE CLASSROOM INSTRUCTION USING THE PROGRAM

A. Explain to the class of students the purpose of the instruction:

   1. Indicate the nature of the subject matter. ______

Checklist for Program Development -- continued

Tasks	Check When Task Completed
2. Specify the method of instruction.	________
3. Other (Specify) ________________	________
B. Train the students in multimedia instructional procedures:	
1. Explain the use of response buttons.	________
2. Orient the learners to the use of the Workbooks.	________
3. Attend to the displays and verbal instructions of explanation and directions.	________
4. Other (Specify) ________________	________
C. Other (Specify) ________________	________
IX. OTHER (Specify) ________________	________

PART II - SECTION B

THE PREPARATION of a PROGRAM SCRIPT and ACCOMPANYING VISUAL DISPLAYS: AN EXAMPLE

THE PREPARATION OF A PROGRAM SCRIPT AND ACCOMPANYING VISUAL DISPLAYS: AN EXAMPLE

Explanatory Note

Program development requires sensitivity to the learning process, understanding of the subject matter, and perceptivity to meaningful instructional presentations. Experience in this writing and designing can be gained only by trial and error carried out in the context of planned strategies. This process is iterative: efforts are made, results noted, and further efforts tried and evaluated. Success cannot be prescribed, but helpful hints can be offered and procedures suggested.

The purpose of the example presented in this section is to illustrate the step-by-step procedures stated in Section A, the Checklist. The illustration is demonstrative; many alternative basic program sequences may be conceived, designed, and developed. It is hoped that the example offered will stimulate and direct the inexperienced programmer in his efforts to apply these procedures of constructing multimedia instructional units. A complete and effective program is achieved by successive approximations. The reader is invited to engage in a reviewing of his own conceptualizations of the teaching-learning process. If he then translates his views into a programed sequence, he will gain insight into significant facilitators of student learning: the individualization of instruction, and the empirical testing of his own notions of programing instructional experiences.

I. PLANNING THE DEVELOPMENT OF THE PROGRAM

A. **Assess the specific needs of classroom instruction which will be fulfilled by development of the program:**

1. **Review, within the perspective of the subject matter curriculum, the particular content area which is being considered for multimedia programing.**

   Consider the various types of equations solved in first year algebra.

2. **Write a brief statement of reasons for considering the proposed content area for programing.**

   The concepts are relevant to the first year algebra curriculum. They are concrete and easily identified and thus lend themselves to initial attempts of a naive programmer. Multimedia provides close control of student learning behavior so essential for making sure the first fundamental concepts are learned well.

3. **List the major topics relevant to the content area which will be programed.**

   The logic of solving equations.

   Use of the associative, distributive, commutative laws to solve equations.

   Linear equations of one variable.

   Linear equations involving symbols of inclusion.

   Simultaneous linear equations.

   Quadratic equations of one variable.

B. Survey the entire set of tasks and jobs to be completed to obtain a gestalt of program development and project planning (i.e., review all items of this Checklist and specify additions where necessary).

> *(The remainder of the Checklist should be read to give an overview as to what is required to a programmer.)*

C. Identify the specific resources and equipment which are available or can be made available.

Personnel

- *Classroom teacher*
- *Programmer*
- *Electronics technician*
- *Artist*
- *Photographer*
- *Teacher Aid/Typist*
- *Program Development Coordinator*

Materials

- *Paper*
- *Pencils and pens*
- *Typewriter*
- *Stencils*
- *Duplicating machine*
- *Materials and equipment for production of visuals*
- *Magnetic recording tape*
- *Slide trays*
- *Method for producing sound cue*
- *Two carousel slide projectors*
- *Two short throw lens for these projectors (focal length 1.4")*
- *Rear projection screen*
- *Overhead projector*
- *Control console and associated equipment (This equipment should be capable of running two slide, or one slide and one motion picture projector simultaneously and should be equipped with a remote stop/start switch.)*
- *Student response button stations (40)*
- *Movie projector*
- *Overhead screen (poorly located--needs changing)*

D. Project the time schedule of the program development by estimating the completion date for each major group of tasks.

> *One unit of learning will be completed the week of 10/6/66, entire program completed by 1/12/67.*

II. PREPARATION OF THE PROGRAM SCRIPT

A. Select one major topic to be programed.

Linear equations of one variable

B. Write down each step of the classroom procedure which is presently used for teaching the selected content learning unit.

Linear equations of one variable are of the forms:

$$\frac{x}{6} - 2 = 3x + 2$$

as opposed to $a + b - 6 = 10 - c$ *in which three variables --* a, b, *and* c *-- occur in the same equation. Also, the variable in a linear equation is not squared, cubed, etc., that is, it is not of a power greater than one. Thus,* $x + 5 = 12$ *is a linear equation of one variable, but* $x^3 + 5 = 12$ *or* $x^2 + 5 = 12$ *is not because* x^2 *and* x^3, *etc., contain powers greater than one.*

A linear equation of the form $x + 5 = 12$ *may be solved by using either the addition or the subtraction axioms previously learned:*

Ex. 1

$$\begin{array}{rcr} x + 5 &=& 7 \\ \underline{\;\;-\,5} && \underline{-5} \\ x &=& 2 \end{array}$$

Reason: equals subtracted from equals

Ex. 2

$$\begin{array}{rcr} 10 &=& x - 3 \\ \underline{+\,3} && \underline{+\,3} \\ 13 &=& x \end{array}$$

Reason: equals added to equals

C. Identify the content objective(s) of the selected topic.

To be able to:

1. *recognize a linear equation of one variable*
2. *solve equations of the following type*

$x + 5 = 10$

$x - 8 = 12$

$12 = x - 6$

$20 = y + 8$

$2x = 6$

$2x - 5 = 6$

$\frac{x}{6} + 3 = 4$

D. Make a judgment, using objective evidence if possible, concerning the level of achievement of the total group and its subgroups for the selected learning unit.

Give a pretest of equations to find what the class knows already about equations.

E. Decide, on the basis of student learning achievement, which type of program will be prepared:

1. Classify the group and/or its subgroups as to the degree of variability of learning ability.

   IQ range is 98-135 and STEP Math score range from 30 to 98 percentile. In general, the class is heterogeneous with regard to learning ability.

2. Decide whether the program will be prepared for the group as a whole or for a particular subgroup or subgroups.

   N.B.: Research evidence strongly suggests that only in rare cases when a total group is very homogeneous will there be justification for preparing one program for total group instruction. Thus a second (or additional) program may be developed simultaneously or scheduled for later development.

   Based upon collected data, the class may be divided into high and low achievers; therefore, two versions will be written, Version A for high achievers and Version B for the low.

F. Partition the outlined learning unit into its parts so that a part defines a set of learning items which can be completed in one unit of daily instruction.

Partition objectives into daily learning units.

First day

1. *Recognize a linear equation of one variable.*
2. *Solve equations of the form:* $x + 5 = 10$; $x - 8 = 12$; $12 = x - 6$; $20 = y + 8$.

Second day

1. *Solve equations of the form:* $2x = 6$; $\frac{x}{7} + 3 = 4$; $2x - 5 = 6$.

G. Select the particular part of the partitioned topic (i.e., the content learning unit) which is to be programed.

All of the daily learning units will be programed.

H. Write the Basic Program Script:

1. Draft the script for the selected unit or units of daily instruction by using the description of the presently used classroom procedures and the list of instructional objectives:

    a. Specify in a logical sequence the behavior(s) which are associated with the objective(s) and which would verify that the objective(s) had been attained, i.e., terminal learning behavior(s).

Version A

---

1. *Discriminate between linear and non-linear equations of one variable.*
2. *Collect evidence to show that discrimination has been learned.*

Etc.

Version B

---

1. *Discriminate between linear and non-linear equations.*
2. *Discriminate between linear equations of one and more variables.*
3. *Associate the word linear with that type of equation.*
4. *Collect evidence that the discrimination has been learned.*
5. *Practice the discrimination.*

Etc.

b. For each of the specified behaviors write one or more subject matter examples of the behavior together with an Instructional comment on the examples.

Version A

Behavior: *Discriminate between linear and non-linear equations of one variable.*

Example	*Comment*
(1) $3x + 1 = 12 - x$ (2) $x^2 + 9 = 27$ (3) $x + y = a + 3$	*Equation (1) is a linear equation. So is equation (3). Equation (1) is the only equation you see which is linear in one variable.*
(1) $5x + 3y = 20$ (2) $5x = 20 - x$ (3) $4x^2 + 3y^3 = 10$ (4) $a + 6 = 30$ (5) $6c - b = 10$	*Use what you see on the left as an example and from this list of equations on the right, copy the ones which are linear in one a variable.*

Behavior: *Collect evidence to show that discrimination has been learned.*

$55y = 10$ $3 + 12 = x$ $x^2 + 9 = 2$ $\frac{6}{y} - 9 = a$	*Which equations are linear in one variable?*

3

H. 1. b. (continued)

Version B

Behavior: *Discriminate between linear and non-linear equations.*

Example	*Comment*
(1) $x^2 + 4 = 8$	*Equation (2) is a linear equation.*
(2) $x + 4 = 8$	*Equation (1) is not.*
$10 - y^3 = 12$	*Write the equation which is linear.*
$b + 7 = 20$	

Behavior: *Discriminate between linear equations of one and two variables.*

(1) $a + b - 6 = d - 4$	*Equation 1 contains three variables, a, b, and d. Equation 2 contains only one variable, a. Obviously, which equation is linear in one variable?*
(2) $12 = a - 2 + 3a$	

Behavior: *Associate the word, linear, with that type of equation.*

$x + 1 = 6$	*We are studying only one type of equation. Look at the example. It contains only one variable and is called what type of equation?*

. . .continued

__Behavior:__ Collect evidence that the discrimination has been learned.

$x^2 + y = 10$ $12 + a - b = 6$ $3y - 6 + 6y = 2y$ $x + y = a^3 + 4$	*(1) Copy the equation or equations which are linear.* *(2) Copy the equation or equations which are linear in one variable.*

__Behavior:__ Practice the discrimination.

	(1) Make up three linear equations of one variable. *(2) Write three non-linear equations.*
Linear of One Variable $y + 6 = 9$ $4x - 19 = 56$ $10 = a + 6 + 3a$	*The first three equations should be like these,*
$y + 6 - x = 9$ $4x + 10y = 19$ $20 + a + b = c$	*but not like these,*
$x^2 + 4 = 10$ $y^3 + 5 = 6y^3$ $6 + a^2 = 12$	*and not like these. The second three should look like these.*

c. **Recheck comments and examples for logical order.**

Comments and examples are in logical order.

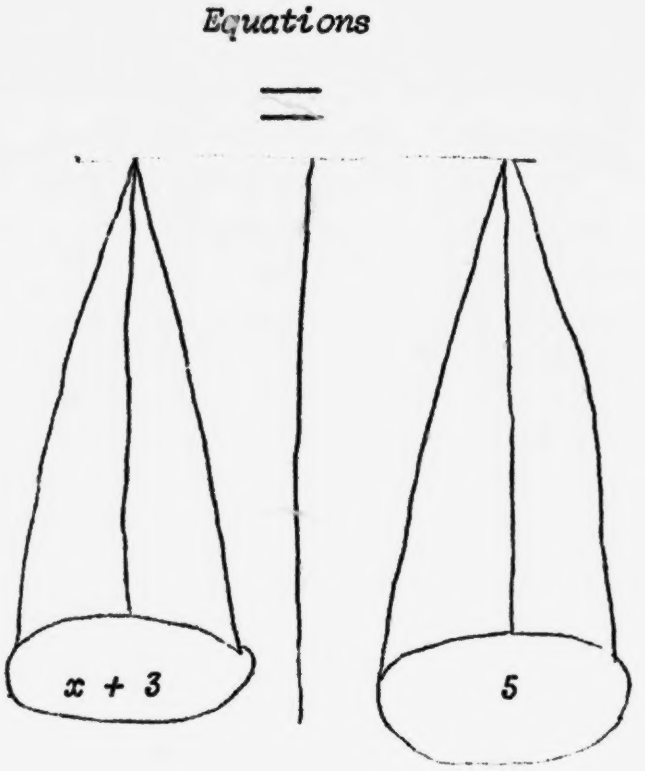
I. Complete the Program Script:

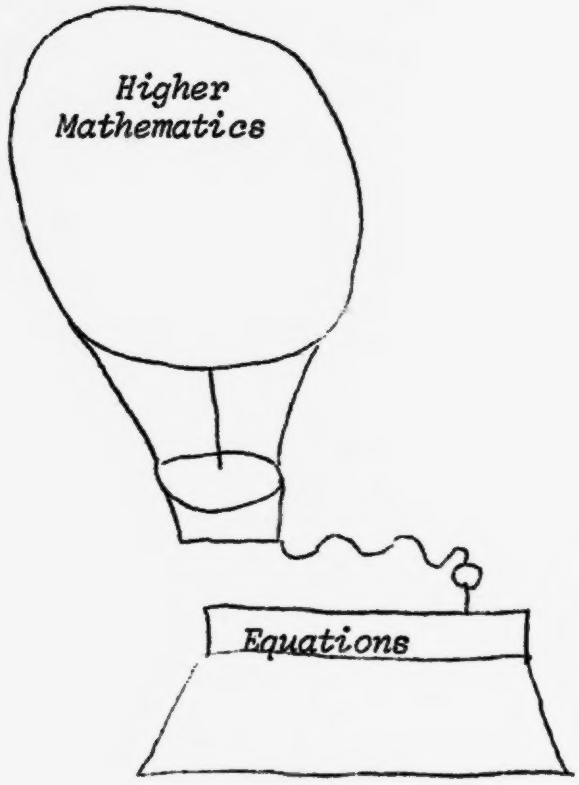
1. **Prepare an introduction to the Basic Program Script to motivate learning by writing an introductory script and sketching needed visual displays to stimulate the learner's interest.**

Introduction

Mathematicians frequently look for more efficient methods of working problems. Many of the problems which you work in arithmetic in seventh and eighth grade may be worked more efficiently if a method involving the solution of equations is used. Solving equations is a basic skill you must learn in order to simplify problem solving and to study advanced mathematics. It is basic in the same way that learning to add, subtract, multiply and divide is basic to the solution of problems you meet in arithmetic.

Today's lesson is designed to introduce you to simple methods of solving equations.



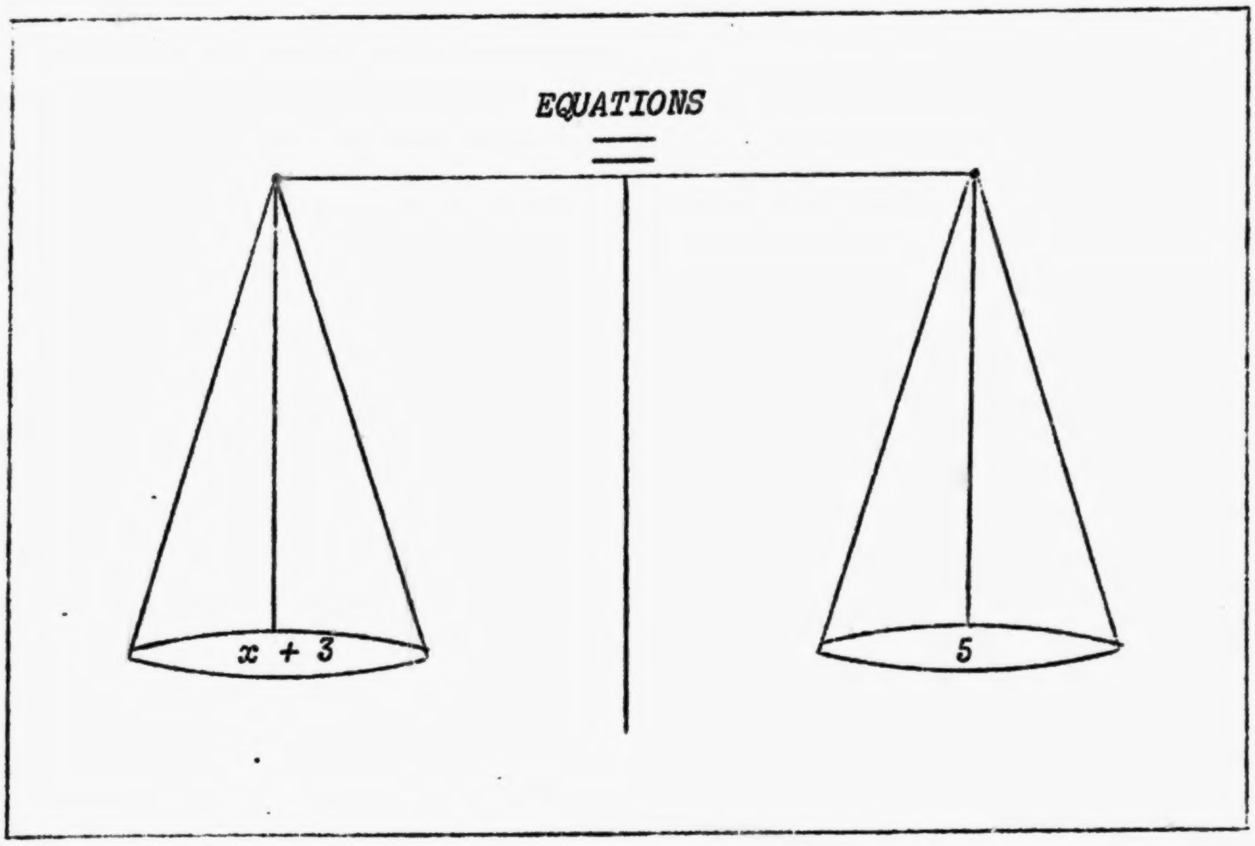


III. PRODUCTION OF THE VISUAL MATERIALS

A. Design the proposed visuals:

3. Prepare detailed plans of each visual:

Program: *Linear Equations of one variable* Slide No. 1



Production Method: *Color Transparency*

Number and letter size: 1) x Standard

2) ___ As large as possible

3) ___ Other ______________ (Specify)

Special effects: *Artist: Use "jazzy" picture of balance, if possible*

Number of copies: 3

Review and Critique of slide (Suggested revisions):

III. A. 3. (continued)

Program: Linear Equations of one variable **Slide No.** 7

$$(I)\ a + b - 6 = d - 4$$

$$(II)\ 12 = a - 2 + 3a$$

Production Method: Kodalith (White numerals on black background)

Number and letter size: 1) x Standard

2) ___ As large as possible

3) ___ Other ______________________
(Specify)

Special effects: Make all letters (a, b, d) red.

Number of copies: 3

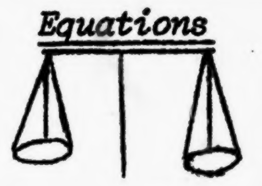
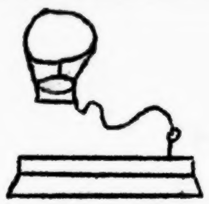
- -

Review and Critique of slide (Suggested revisions):

IV. COORDINATION OF THE PROGRAM SCRIPT AND THE VISUAL DISPLAYS

PROGRAMING WORKSHEET

Program Title: *Linear equations of one variable -- first instructional unit -- Version B (Low Ability)* **Page** 1

SCREEN		RECORDED SCRIPT and CUE POINTS	STUDENT WORKSHEET
Left	Right		
(1) 		*(1) Mathematicians frequently look for more efficient methods of working problems. Many of the problems which you work in arithmetic in seventh and eighth grade may be worked more efficiently if a method involving the solution of equations is used. Solving equations is a basic skill you must learn in order to simplify problem solving and to study higher mathematics.*	
	(2) 	*(2) It is basic in the same way that learning to add, subtract, multiply, and divide is basic to the solution of problems you met in arithmetic.*	

IV. (continued)

Program Title: *Linear equations of one variable -- first instructional unit -- Version B (Low Ability)* Page 2

SCREEN Left	SCREEN Right	RECORDED SCRIPT and CUE POINTS	STUDENT WORKSHEET
(3) (I) $x^2 + 4 = 8$ (II) $x + 4 = 8$	(4) *Blank*	*Today's lesson is designed to introduce you to simple methods of solving equations. We shall begin with this slide. (3) (4) First you should learn to recognize a linear equation in one variable since this is the type of equation you will solve first. Equation (II), $x + 4 = 8$, is a linear equation in one variable but equation (I) is not. Notice the difference between the two. (Pause)*	
	(5) $10 - y^3 = 12$ $b + 7 = 20$ $a + 8 = a^2 + 9$	*(5) Look at these equations on the right. Decide which one is linear and write it for answer one on your worksheet. (Sound effect indicating problem to be worked)*	1. *Which equation is linear?* *ANSWER* ____________

IV. (continued)

Program Title: *Linear equations of one variable -- first instructional unit -- Version B (Low Ability)* Page 3

SCREEN Left	SCREEN Right	RECORDED SCRIPT and CUE POINTS	STUDENT WORKSHEET
(6) $b + 7 = 20$		(6) *You should have written this.*	
(7) (I) $a + b - 6 = d - 4$ (II) $12 = a - 2 + 3a$	(8) *Blank*	(7) (8) *Now notice these two equations. Equation (I) contains three variables,* a, b, d, *while equation (II) contains only the single variable,* a. *So, which equation is linear in one variable?* *(Sound effect)*	
(9) $12 = a - 2 + 3a$		(9) *The equation* $12 = a - 2 + 3a$ *is the one you should have copied. It is linear and contains only one variable.* *Etc.*	2. *Copy the equation which contains one variable.* ANSWER ____________

PART III

SELECTED PROGRAMS

A GENERAL INTRODUCTION TO THE PROGRAMS

This section focuses on actual scripts developed for and used in the experimentation. For each of three topics, an original or revised program is presented in two versions -- A for high ability and B for low ability; introductory remarks to each presentation describe a) the revisional procedures that preceded and/or followed the program presented herewith, and b) the general difference(s) between ability versions. For the fourth topic, Graphing of Inequalities, the entire sequence of programs for both A and B versions is presented; by carefully examining these six scripts, the reader will be able to detect both major and minor changes among programs and between versions.

An attempt has been made to present the programs in a format which allows the reader to see the interrelationships of the script, visual aids, and student workbooklets. The following discussion of the format is applicable to all programs.

The script, which was recorded on magnetic tape, is presented in the first column of the right-hand page. The asterisks (*) within a script indicate the occurrence of a "cricket" sound-effect, recorded with the text; this sound directed the student to work a problem in his answer booklet. The substance of the booklet, i.e., the worksheet, is presented in the second column of the right-hand page.

The left-hand page represents the screen, divided into two parts; images may be projected on both simultaneously. A slide appears on the screen when the number corresponding to it appears in the script. This slide remains on the screen until the next slide on that half of the screen comes on. A blank slide indicates no visual on that half of the screen. The slides represented herewith convey the main idea of the slides actually used in the experimentation. Printing limitations have necessitated the omission of some detail, as well as color; the few extremely complex slides and film clips are described, rather than reproduced.

Following the versions of each program is the appropriate measure, Y_1 accompanying the original program (p'), Y_2 accompanying the first revision (p''), and Y_3 accompanying the second revision (p'''). Where-ever possible, the answers to test items are shown; distribution of points awarded for correct responses is given at the end of each measure.

SCIENTIFIC NOTATION

INTRODUCTION TO THE SCIENTIFIC NOTATION PROGRAM

In this program, one general concept was taught: how to write numbers greater than 1 in scientific notation.

The original high version p'A taught the concept in two steps: 1) an example of a number written in scientific notation was presented, but no explanation of the process involved was given; and 2) the students worked two examples involving scientific notation. One example required the completion of statements, such as

$$3{,}456 = 3.456 \times \underline{\qquad} = 3.4 \times 10^{\_},$$

while the second asked the student to write the diameter of the Milky Way in scientific notation.

The only problem encountered in the p'A version involved writing the correct power of 10. Many students equated the power of 10 with the number of zeros in the number to be written in scientific notation. For example, very often 4200 was written as 4.2×10^2, rather than 4.2×10^3. Discrimination learning was accomplished in the first revision, p''A, by merely asking the student to compare the exponent of 10 in the expression

$$4200 = 4.2 \times 1000 = 10^3$$

with the number of zeros in 4200 and with the number of zeros in 1000. No further revision of this program was required; thus, p''A and p'''A were identical.

The low version, B, was presented in smaller, more explicit steps which required more frequent student responses. In the original version, p'B, the general concept was taught in three parts; for example, 42000, written in scientific notation (4.2×10^3), consists of 1) a number between 1 and 10, 2) the sign x, and 3) 10 raised to the correct power.

Two main problems occurred in the original version. As in the high version, the proper exponent of 10 was confused with the number of zeros in the original number to be written in scientific notation. This difficulty was corrected with the introduction of discrimination learning; examples were introduced in which the student compared exponents with zeros (as explained above for the high version) and found mistakes in incorrect expressions. The second problem occurred when students had difficulty writing the first number in scientific notation between 1 and 10; this problem was corrected by introducing exercises in which the student was required to place decimal points in numbers so that the resulting numbers were between 1 and 10. In both p''B and p'''B, attempts to eliminate these problems involved the presentation of varied examples.

SCIENTIFIC NOTATION:

FIRST REVISION p'' A

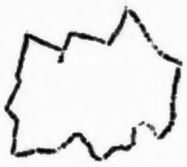
SCREEN

Left Half

Right Half

(1)

12 GRAMS OF CARBON



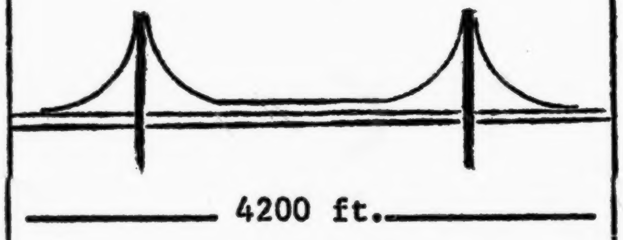
602,300,000,000,000,000,000,000
ATOMS

(B)

Blank Slide

(17)

GOLDEN GATE BRIDGE



(23)

Scientific Notation

$$4200 = 4.2 \times 1000$$
$$= 4.2 \times 10^3$$

RECORDED SCRIPT	WORKSHEET
(1) (B) This is a picture of 12 grams of carbon. The number of atoms in 12 grams of carbon is written below the picture. This number, as you can see, is very large. Sometimes, scientists must deal with numbers such as this and thus need shorter methods of writing them. In this lesson you will learn how scientific notation may be used to write large numbers in a considerably briefer form.	
(17) The golden gate bridge is 4200 feet long. This length written in scientific notation may be written this way. (23) Study this example. Compare the exponent of 10, that is, the 3 with the number of zeros in 4200 and the number of zeros in 1000. Fill in the blanks on your worksheet for problem 1. *	1. Complete these statements a.) 3,456 = 3.4 x ___ = 3.4 x 10___ b.) 540,000 = ___ x 100,000 = ___ c.) 498 = 4.98 ___ 100 = 4.98 _____ (Press button D when you have completed answers.)

SCREEN

Left Half

(26)

(a) $3,456 = 3.4 \times 1000 = 3.4 \times 10^3$

(b) $540,000 = 5.4 \times 100000 = 5.4 \times 10^5$

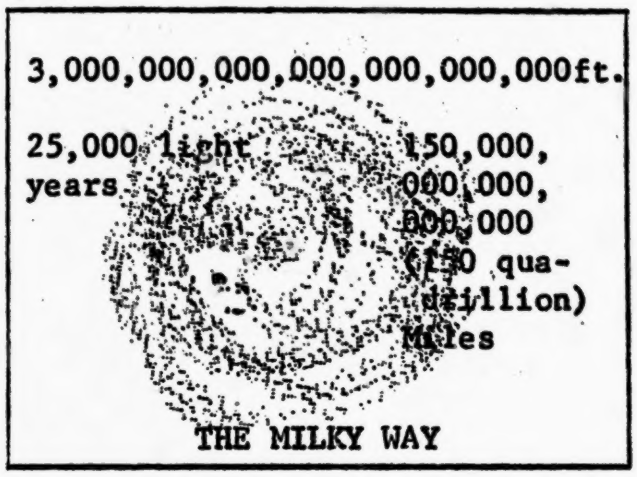
(c) $498 = 4.98 \times 100 = 4.98 \times 10^2$

Right Half

(B)

Blank Slide

(29)



3,000,000,000,000,000,000,000ft.

25,000 light years

150,000, 000,000, 000,000 (150 quadrillion) Miles

THE MILKY WAY

(30)

3×10^{21} feet

RECORDED SCRIPT	WORKSHEET
(26) (B) Here are the correct answers. Check your work carefully. Find and understand your mistakes.	
(29) The diameter of the Milky Way is indicated here in feet. Write this number in scientific notation. *	2. Write the diameter of the Milky Way in scientific notation. Answer: ____________________ (Press button C when you have completed your answer.)
(30) Here is the correct answer.	
You will now be given a short quiz to determine what you have learned.	

SCIENTIFIC NOTATION:

FIRST REVISION p'' B

14/

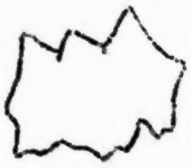
SCIENTIFIC NOTATION - p'' B

SCREEN

Left Half	Right Half

(1)

12 GRAMS OF CARBON



602,300,000,000,000,000,000,000
ATOMS

(B)

Blank Slide

(2)

Scientific Notation

6.023×10^{23}

(3)

(a) 1 and 10

(b) 1 and 6

(c) 600 and 700

(d) 50 and 80

RECORDED SCRIPT	WORKSHEET
(1) This is a picture of 12 grams of carbon. The number of atoms in 12 grams of carbon is written below the picture. This number, as you can see, is very large. Sometimes scientists must deal with numbers such as this and need shorter methods of writing them. In this lesson you will learn how scientific notation may be used to write large numbers in a considerably briefer form. The number of atoms in 12 grams of carbon is 6,023 followed by how many zeros?*	1. How many zeros follow the 6023? ANSWER ____________________ (Press C when finished.)
There are 20 zeros following 6023. Count them again if you miscounted.	
(2) Here is the number 6023 followed by 20 zeros written in scientific notation.	
Look at the scientific notation. The first numeral is written in red. This red numeral is between which of the following two numbers? (3)	2. Write the correct answer.
Of course, 6.023 is between 1 and 10 so you should have written 1 and 10.	

SCREEN

Left Half

(7)

Scientific Notation

602,300,000,000,000,000,000,000

$= 6{,}023 \times 10^{23}$

$460{,}000{,}000{,}000 =$

4.6×10^{11}

Right Half

(B)

Blank Slide

(8)

5,623,000,000

RECORDED SCRIPT	WORKSHEET
(7) (B) Look at these two numbers and their form in scientific notation.	
You should learn that to write some numbers in scientific notation, write first a numeral like the ones indicated in red. These numerals, as you probably suspect, are between what two numbers?*	3. The red numbers are between what two numbers?
The answer is 1 and 10. 6.023 and 4.60 are both between 1 and 10. The first numeral you write in scientific notation is between 1 and 10.	
(8) Consider the number 5,623,000,000. If you were to write this in scientific notation, between what two numbers would you place the decimal point?	4. Between what two numbers would you place the decimal point?
To write this number in scientific notation the decimal point would be placed between the 5 and the 6.	

SCIENTIFIC NOTATION - p'' B

SCREEN

Left Half

Right Half

(9)

$5,623 \times 10^6$

56.23×10^8

5.623×10^9

562.3×10^7

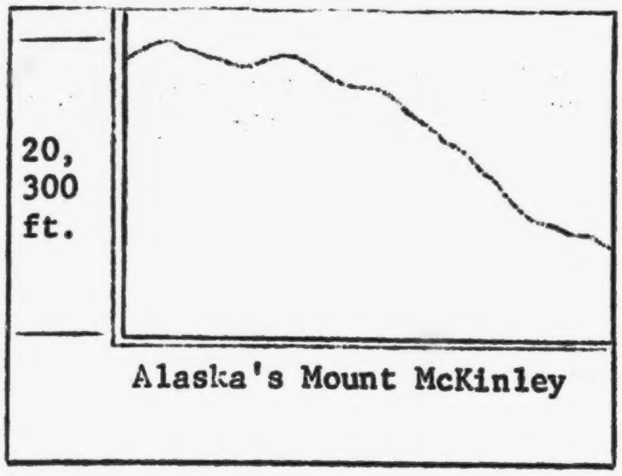
SCIENTIFIC NOTATION:

FIRST REVISION p'' A

(10)

5.623×10^9

(11)


20,
300
ft.
Alaska's Mount McKinley

(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
(9) Which one of the numbers on the right is the correct method of writing this number in scientific notation?*	5. Which is correct scientific notation?
(10) Here is the correct answer. Notice that the first numeral is 5.623, and is between 1 and 10.	
(11) (B) Mt. McKinley is twenty thousand three hundred feet high.	

SCREEN

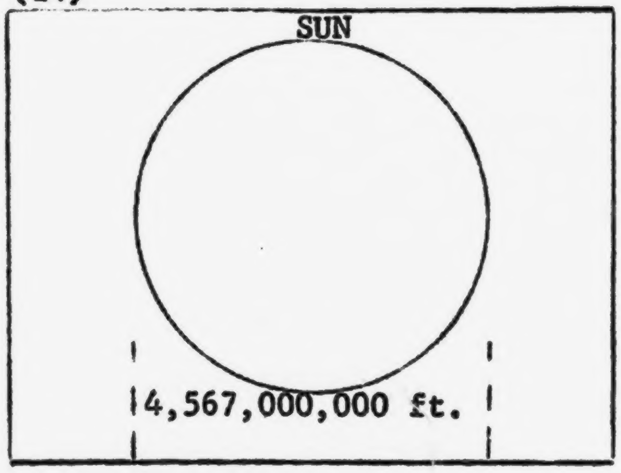
Left Half | **Right Half**

(12)

Scientific Notation

2.03×10^4 feet

(14)


SUN
4,567,000,000 ft.

(B)

Blank Slide

(15)

$4.567 + 10^9$

$4.567 - 10^9$

4.567×10^9

$4.567 \div 10^9$

RECORDED SCRIPT	WORKSHEET
This height written in scientific notation is this. (12)	
Notice the green part.	
A number written in scientific notation first contains a number between 1 and 10 and then a times sign.	
(14) (B) The diameter of the sun is 4 billion 5 hundred sixty-seven million feet. What is the correct method of writing this number in scientific notation?* (15)	6. Write the correct answer for the diameter of the sun in scientific notation.

SCREEN

Left Half

Right Half

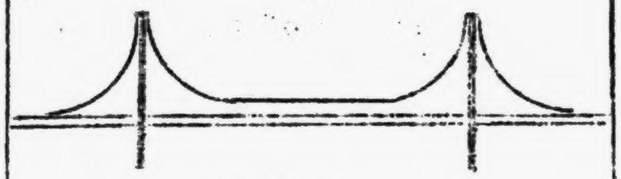
(16)

Scientific Notation

4.567×10^9

(17)

GOLDEN GATE BRIDGE



4200 ft.

(B)

Blank Slide

(18)

4.2 x 10

4.2 x 100

4.2 x 1,000

4.2 x 10,000

RECORDED SCRIPT	WORKSHEET
(16) This is the correct answer. 4.567 should be followed by a times sign.	
(17) (B) The golden gate bridge is 4,200 feet long.	
The number 4,200 is which of the following? (18) If you have to, multiply each answer given until you find the correct one.*	7. Which is 4200?

SCREEN

Left Half | **Right Half**

(19)

4,200 = 4.2 x 1000

Solution: 4.2
x 1000
4200.0

(20)

560 = 5.6 x ______

56.0 = 5.6 x ______

560,000 = 5.6 x ______

(B)

Blank Slide

(21)

500 = 5.6 x 100

56.0 = 5.6 x 10

560,000 = 5.6 x 100,000

RECORDED SCRIPT	WORKSHEET
(19) 4,200 is 4.2 x 1000 (Notice the solution).	
(20) (B) Complete exercise 8 on your answer sheet. (Write 10, 100, 1000, 10000, etc. for answers.) Multiply your final answers to make sure they are correct.*	8. a.) 560 = 5.6 x __________ b.) 56.0 = 5.6 x __________ c.) 560,000 = 5.6 x __________
Here are the correct answers. (21)	

SCREEN

Left Half

(23)

Scientific Notation

$$4200 = 4.2 \times 1000$$

$$= 4.2 \times 10^3$$

Right Half

(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
(23) (B) If the length of the golden gate bridge is written in scientific notation, it may be written this way.	
Now I'm going to ask some questions which may seem easy but aren't necessarily so. Look at the 4,200. How many zeros are in 4,200?* Look at the 1000. How many zeros are in the 1000?* Of course, there are 2 zeros in 4,200; however there are three zeros in 1000. Now notice the exponent of the power of 10, that is, the 3. Is this exponent of ten the same as the number of zeros in 4,200?* Is the exponent of ten the same as the number of zeros in the 1000?*	9. a.) How many zeros in 4200? b.) How many zeros in 1000? c.) Is the exponent the same number of zeros in 4200? (Yes or No) d.) Is the exponent the same number of zeros in 1000? (Yes or No)
You should have written first "No" and then "Yes". The exponent of 10 is not the number of zeros in 4,200, but is the number of zeros in 1000.	
Use 4200 as a model and write the number 53,000 in scientific notation. This number appears in problem 10 of your answer sheet.	10. Write 53,000 in scientific notation by completing the following: 53,000 = 5.3 x ____________ = 5.3 x ____________

SCREEN

Left Half | Right Half

(24)

$53{,}000 = 5.3 \times 10{,}000$

$= 5.3 \times 10^4$

(25)

Scientific Notation

(a) 3,456 = 3.4x __ = 3.4x $10^{\_}$

(b) 540,000 = ___ x 100000 = ___ x 10^5

(c) 498 = 4.98 __100 = 4.98 ___10^2

(B)

Blank Slide

(26)

(a) 3,456 = 3.4x1000 = 3.4x 10^3

(b) 540,000 = 5.4 x 1000000 = 5.4 x 10^5

(c) 498 = 4.98 x 100 = 4.98x 10^2

RECORDED SCRIPT	WORKSHEET
(24) Here is the correct answer.	
(25) (B) Complete a, b and c by copying each statement completely and filling in the blanks correctly.*	11. a.) ____________________ b.) ____________________ c.) ____________________
(26) Here are the correct answers. Check your work carefully and understand your mistakes.	

SCIENTIFIC NOTATION - p'' B

SCREEN

Left Half

(27)

(a) 90,765

(b) 1,870,000

Right Half

(B)

Blank Slide

(28)

(a) 9.0765×10^4
Solution: $90{,}765 = 9.0765 \times 10{,}000 = 9.0765 \times 10^4$

(b) 1.87×10^6
Solution: $1{,}870{,}000 = 1.87 \times 1{,}000{,}000 = 1.87 \times 10^6$

RECORDED SCRIPT	WORKSHEET
(27) (B) Write these two numbers in scientific notation.*	12. a.) 90,765 = ______________ b.) 1,870,000 = ______________
(28) You should have written this.	

SCIENTIFIC NOTATION:
MEASURE Y_2

	ANSWER:
1. How is 70,400,000 written in scientific notation? a.) 70.4×10^9 b.) 704×10^8 c.) 7.04×10^{10} d.) $.704 \times 10^{11}$ e.) all of the above	1. c
2. How is 532,000,000 written in scientific notation? a.) $5.32 \div 10^8$ b.) 53.2×10^7 c.) $53.2 \div 10^7$ d.) 5.32×10^8 e.) none of the above	2. d
3. How is 969,000,000 written in scientific notation? a.) 96.91×10^7 b.) $96.91 \div 10^7$ c.) $10^8 \times 9.691$ d.) 9.691×10^8 e.) none of the above	3. d
4. How is 42,000,000 written in scientific notation? a.) 42×10^9 b.) $42,000 \times 10^6$ c.) $.42 \times 10^{11}$ d.) 4.2×10^{10} e.) all of the above	4. d
5. The distance between two stars is 560,000 miles. Write this number in scientific notation.	5. 5.6×10^5

SCIENTIFIC NOTATION: MEASURE Y_2

Write each number in scientific notation:

		ANSWER:
6.	9880	6. 9.88×10^3
7.	876,000	7. 8.76×10^5
8.	252	8. 2.52×10^2
9.	567,000,000,000,000	9. 5.67×10^{14}
10.	50	10. 5.0×10

SYSTEM WITHOUT NUMBERS

INTRODUCTION TO THE SYSTEM WITHOUT NUMBERS PROGRAM

This programed lesson teaches four operations: R_1, R_2, and R_3 are performed by the rotation of a rectangle about one of three axes, while R_0 is an operation in which no rotation occurs. The four operations, performed by each student with the rectangle provided him, comprise a system without numbers. These operations were used with the sysbol ⓕ, "is followed by," to teach that the system possessed the commutative and associative properties of the rational number system.

Consider the A (high) version and its revisions. In p'A the four operations were presented on film. After the presentation, the students were asked to perform each operation and to write the results. The symbol ⓕ was taught by merely defining it and asking each student to work examples in which the symbol appeared. These examples illustrated the associative and commutative laws; each student responded to the questions requiring that he write the words associative or commutative. It was found that not all students started each operation with the rectangle in the same position as was illustrated in the film and they were unable to learn the operations in a single presentation. The instruction "Write each result on your worksheet for problem 1" was confusing even though blank rectangles were provided.

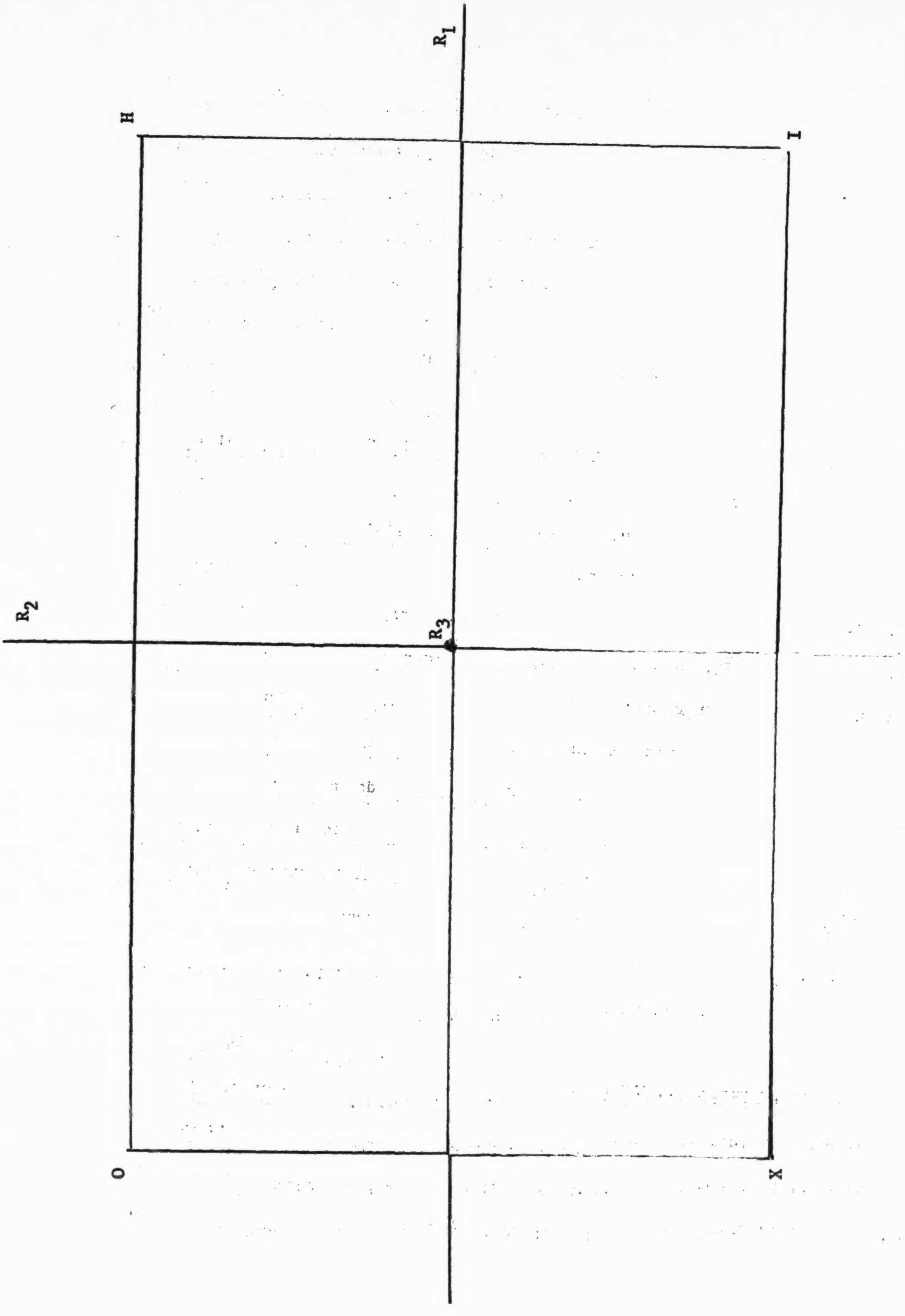
The first revision, p''A, emphasized that the student should begin each operation with the rectangle in its original position. A practice period was introduced after the presentation of each operation; after the four were presented, the student again performed the four operations and wrote the results. The instructions for writing the results were revised to read "Indicate each result by labeling the four corners of the rectangle in problem 1." The commutative and associative laws were mentioned in the script, but no written responses of the words, i.e., associative and commutative, were required.

Since these revisions involved no learning problems, the final revision, p'''A, involved shortening the program so that the material could be learned in less time. The explanation that matching corners indicated correct original position of the rectangle was eliminated; instead, a picture of the rectangle in what was called "its original position" was presented briefly. The practice of each operation after its visual presentation was retained, but additional practice of these operations was eliminated.

The B, or low, version taught the same concepts as the high version. However, material was presented in much smaller steps, i.e., less information was given at one time, students' attention was directed by more explicit and detailed instructions, and practice was used more frequently.

The first revision, p''B, consisted of the following: 1) it was pointed out that the rectangle is not picked up when performing operation R_0, 2) the student was asked to write his results "by labeling the four corners of the rectangle in problem 3," and 3) explicit information was provided to clarify that the rectangle was not returned to its original position after performing R_1.

The only change required for Version p'''B was to revise the instructions again for performing R_1 and then R_2. Many students continued to return the rectangle to its original position after performing R_1. The instructions were made more directive: "After you have performed operation R_1, stop. Leave your rectangle in the final position."


R$_1$
H
I
R$_2$
R$_3$
O
X

SYSTEM WITHOUT NUMBERS:

SECOND REVISION p''' A

SYSTEM WITHOUT NUMBERS - p''' A

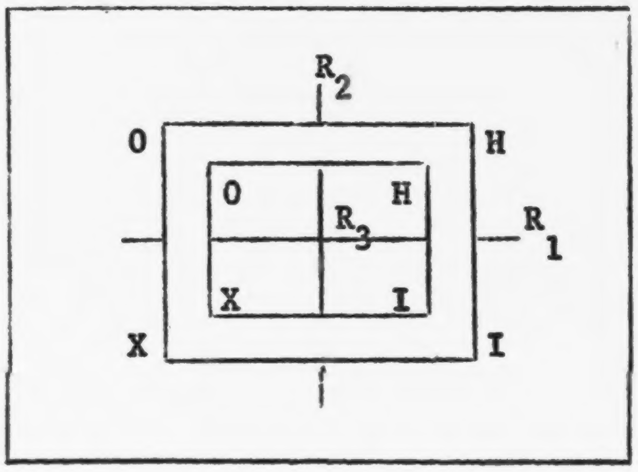
SCREEN

Left Half | Right Half

(0)

SYSTEM
WITHOUT
NUMBERS

(1)


R_2
O
H
O
H
R_3
R_1
X
I
X
I

(B)

Blank Slide

RECORDED SCRIPT

WORKSHEET

(0) You may be surprised to find that not all algebra is concerned with numbers. There are mathematical systems with important applications in the scientific world which contain elements other than numbers. Most of these are too complicated for beginners, but we are going to look at an example to give you an idea of what is meant by a mathematical system which contains no numbers.

(1) Each of you has on your worksheet a diagram like the one on the screen. You also have a rectangle which fits the diagram. The rectangle as you see it is in its original position. Place your rectangle in the original position. You'll notice that there are three symbols in red on the diagram, R_1, R_2, and R_3. Each of these R's represents a certain operation. You will now learn to perform each of these operations. (B)

SYSTEM WITHOUT NUMBERS - p''' A

SCREEN

Left Half	Right Half
(M) Movie Clip: Person performs operation R_1 with a large cardboard rectangle.	
(M) Movie Clip: Person performs operation R_2 with a large cardboard rectangle.	
(M) Movie Clip: Person performs operation R_3 with a large cardboard rectangle.	

RECORDED SCRIPT **WORKSHEET**

To perform operation R_1, rotate the rectangle about the R_1 axis 180°. Watch this being done in motion pictures. (M) Now for practice, you try what you just saw.

To perform operation R_2, start from the original position and rotate the rectangle about the R_2 axis 180°. Watch this being done in motion pictures. (M) Practice operation R_2 as in the movie.

Operation R_3 is performed by rotating the rectangle about the R_3 axis. Watch this. (M) Now try R_3 a few times.

SYSTEM WITHOUT NUMBERS - p''' A

SCREEN

Left Half

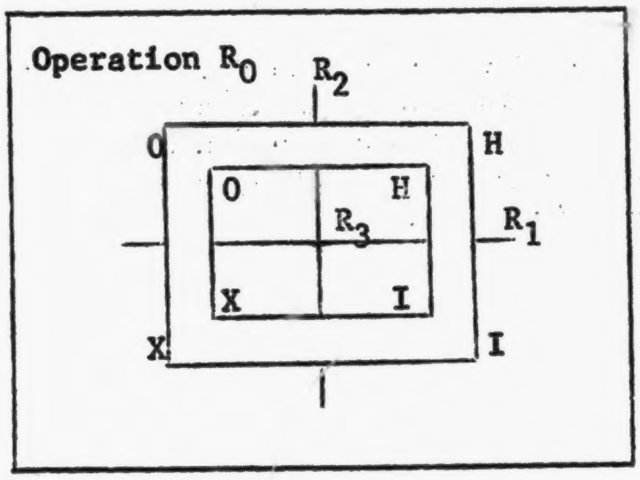
Right Half

(10)

R_0

No Operation

(11)


Operation R_0
R_2
O
H
O
H
R_3
R_1
X
I
X
I

(18)

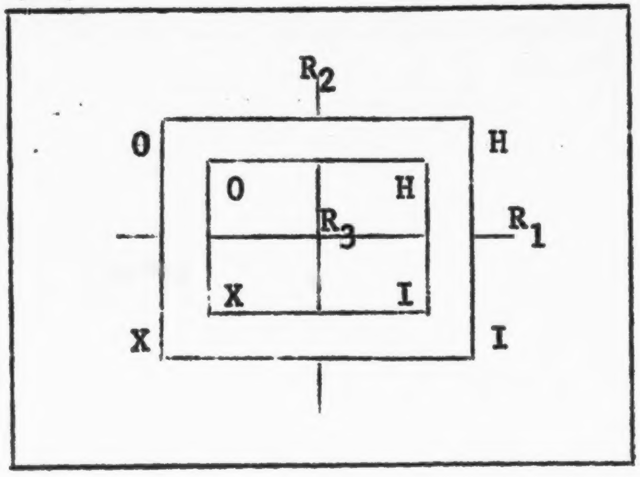
$R_3 \textcircled{f} R_1 = \underline{?}$

followed by

(B)

Blank Slide

(15)


R_2
O
H
O
H
R_3
R_1
X
I
X
I

RECORDED SCRIPT	**WORKSHEET**
We shall let operation R_0 indicate no operation. (10) This means that the rectangle is not moved. It stays in this position. (11)	
See this expression. (18) (B)	
The "f" with a circle around it means "followed by." The expression is read R_3 followed by R_1. To work this problem, put your rectangle in its original position. (15) Everyone perform operation R_3	

SCREEN

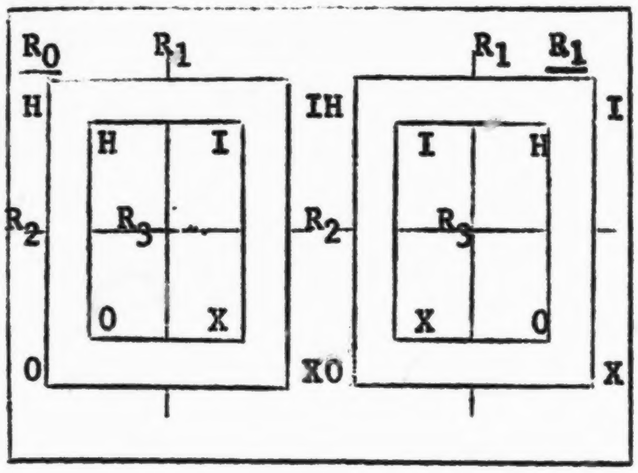
Left Half

Right Half

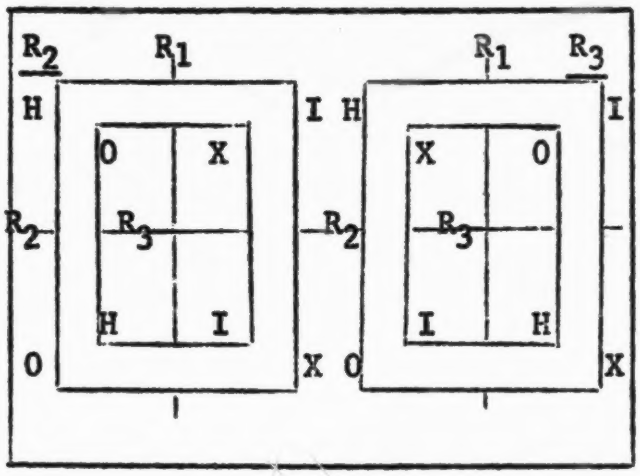
(B)

Blank Slide

(16A)


R_0
R_1
R_1
R_1
H
IH
I
H
I
I
H
R_2
R_3
R_2
R_3
O
X
X
O
O
XO
X

(16B)


R_2
R_1
R_1
R_3
H
I H
I
O
X
X
O
R_2
R_3
R_2
R_3
H
I
I
H
O
X O
X

(19)

$$R_3 \textcircled{f} R_1 = R_2$$

(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
(B) Now without returning the rectangle to the original position, perform operation R_1.	
Compare your rectangle with what you see on the screen. (16A) (16B) Thus, R_3 followed by R_1 is the same as which of the four operations?	1. R_3 ⓕ R_1 is the same as ________________
You should have written R_2. (19) (B)	

SCREEN

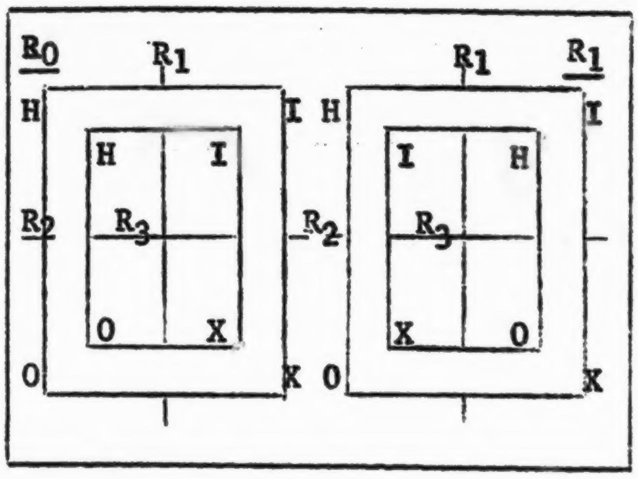
Left Half

Right Half

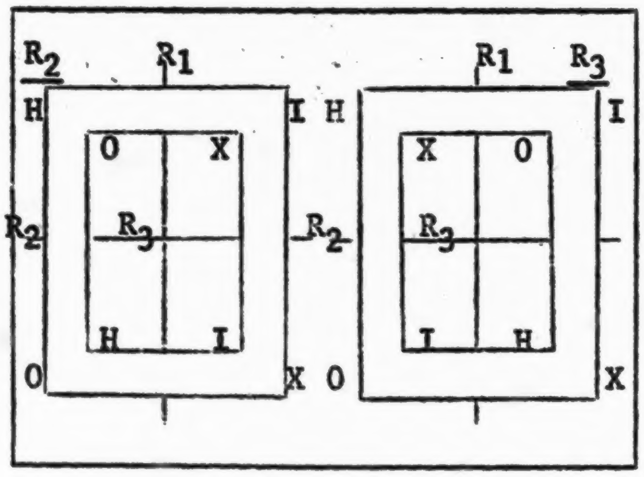
(B)

Blank Slide

(16A)


R0
R1
R1
R1
H
I H
I
H
I
I
H
R2
R3
R2
R3
O
X
X
O
O
X O
X

(16B)


R2
R1
R1
R3
H
I H
I
O
X
X
O
R2
R3
R2
R3
H
I
I
H
O
X O
X

(20)

10

(a) $R_0 \textcircled{f} R_1 = R_1$

(b) $R_2 \textcircled{f} R_3 = R_1$

(c) $R_3 \textcircled{f} R_2 = R_1$

(d) $R_1 \textcircled{f} R_1 = R_0$

(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
(B) Use your rectangle to work all of problem 2.	
(16A) (16B) Refer to the operations on the screen and write R_0, R_1, R_2, R_3 for answers.*	2. (a) $R_0 \textcircled{f} R_1 =$ __________ (b) $R_2 \textcircled{f} R_3 =$ __________ (c) $R_3 \textcircled{f} R_2 =$ __________ (d) $R_1 \textcircled{f} R_1 =$ __________
Here are the correct answers (20) (B) Notice that "b" and "c" illustrate the commutative law.	

SCREEN

Left Half

Right Half

(22)

(a) $(R_1 \textcircled{f} R_2) \textcircled{f} R_3 =$

(b) $R_1 \textcircled{f} (R_2 \textcircled{f} R_3) =$

(23)

(a) $(R_1 \textcircled{f} R_2) \textcircled{f} R_3 = R_0$

(b) $R_1 \textcircled{f} (R_2 \textcircled{f} R_3) = R_0$

(24)

R_0, R_1, R_2, R_3

$\textcircled{f}$

(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
(22) Here are two problems, each involving two operations. Notice that each problem is grouped differently by using parentheses. To work these problems, perform the operations in the parentheses first. Write the correct answers.	3. (a) $(R_1 \textcircled{f} R_2) \textcircled{f} R_3 =$ ________ (b) $R_1 \textcircled{f} (R_2 \textcircled{f} R_3) =$ ________
(23) You should have gotten R_0 for each answer. This illustrates the associative law.	
(24) (B) So our set of four elements -- R_1, R_2, R_3, and R_0-- and the operation "followed by" have the commutative and associative properties just as our familiar system of integers has. We could investigate other properties which we know integers have, to see if they are properties of this system also.	

SYSTEM WITHOUT NUMBERS:

SECOND REVISION p''' B

SYSTEM WITHOUT NUMBERS - p''' B

SCREEN

Left Half

Right Half

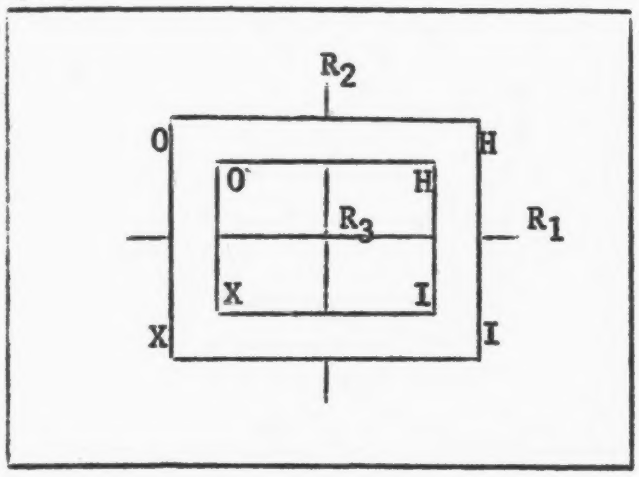
(0)

SYSTEM

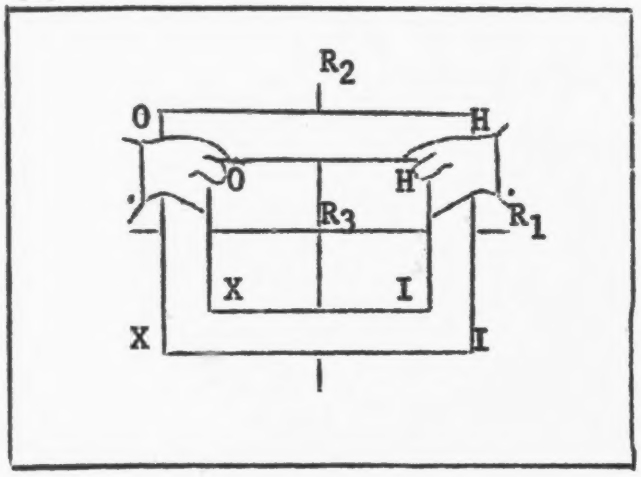
WITHOUT

NUMBERS

(1R)


R₂
O
H
O
H
R₃
R₁
X
I
X
I

(2)


R₂
O
H
O
H
R₃
R₁
X
I
X
I

RECORDED SCRIPT **WORKSHEET**

(0) You may be surprised to find that not all algebra is concerned with numbers. There are mathematical systems with important applications in the scientific world which contain elements other than numbers. Most of these are too complicated for beginners, but we are going to look at an example to give you an idea of what is meant by a mathematical system which contains no numbers.

(1R) Each of you has on your worksheet a diagram like the one on the screen. You also have a rectangle which fits the diagram. The rectangle as you see it is in its original position. This means that the H's, X's, Q's and I's in the four corners match. Place your rectangle in the original position. You'll notice that there are three symbols in red on the diagram, R_1, R_2, R_3. Each of these R's represents a certain operation. You will now learn to perform each of these operations.

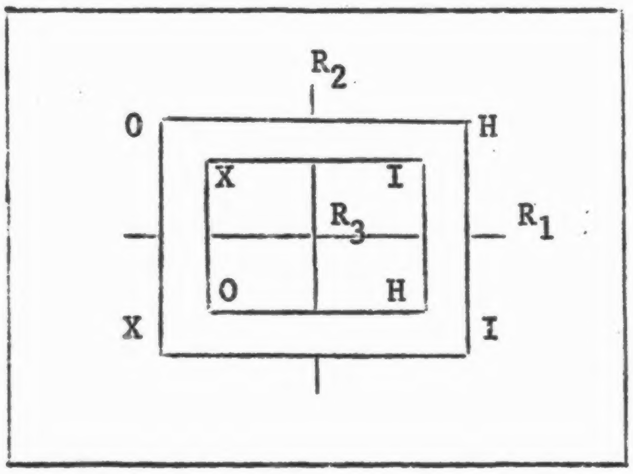
(2) Each of you perform operation R_1 by doing this. With thumb and forefinger of each hand, grasp corners O and H of the rectangle. Now turn the rectangle over.

SCREEN

Left Half

Right Half

(2A)


R2
O
H
X
I
R3
R1
O
H
X
I

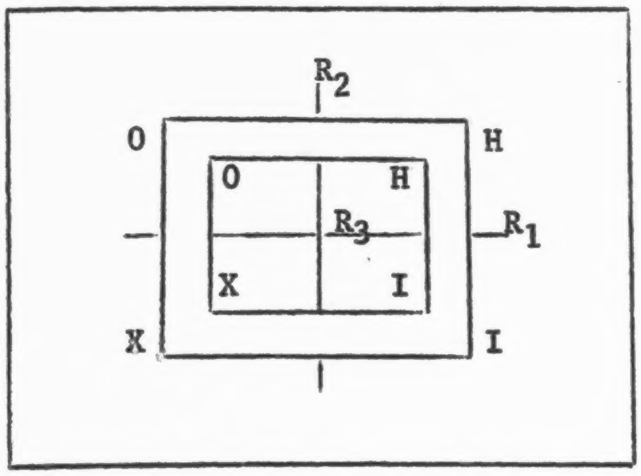
(M)

Operation R_1 being performed in motion pictures.

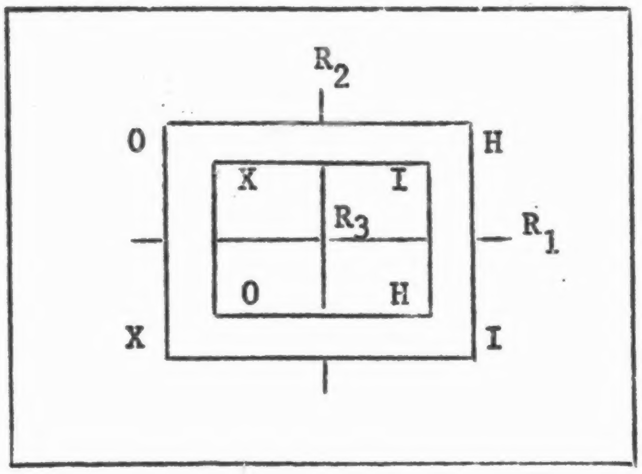
(B)

Blank Slide

(1)


R2
O
H
O
H
R3
R1
X
I
X
I

(2A)

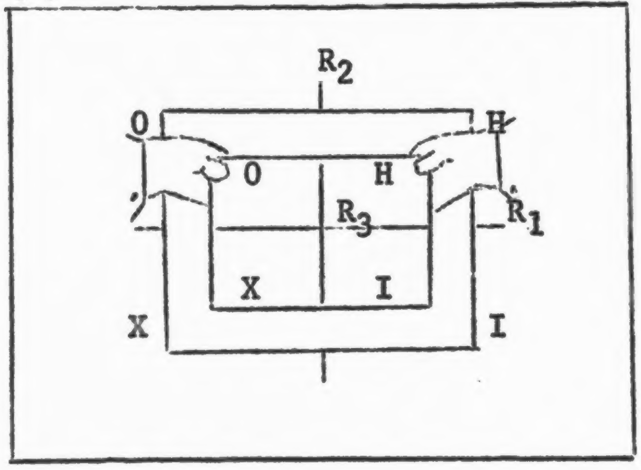

R2
O
H
X
I
R3
R1
O
H
X
I

RECORDED SCRIPT	WORKSHEET
(2A) If you performed the operation correctly, the result looks like this.	
(B) Watch this being done on film. (M)	
(1) Put the rectangle in its original position and do this operation again. Make sure you get this result. (2A)	

SCREEN

Left Half — **Right Half**

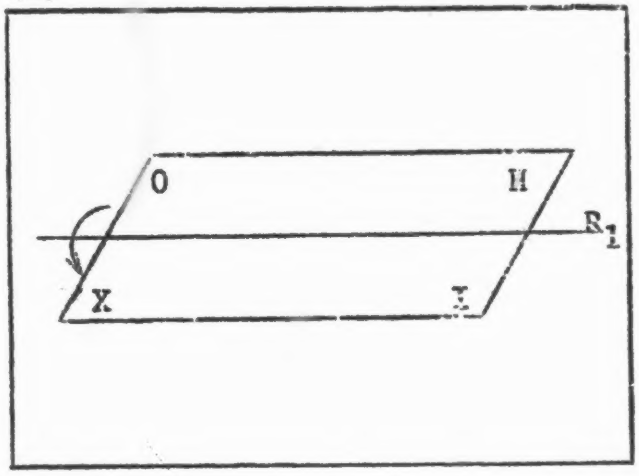
(2)


R2
O
H
O
H
R3
R1
X
I
X
I

(B)

Blank Slide

(3)



(B)

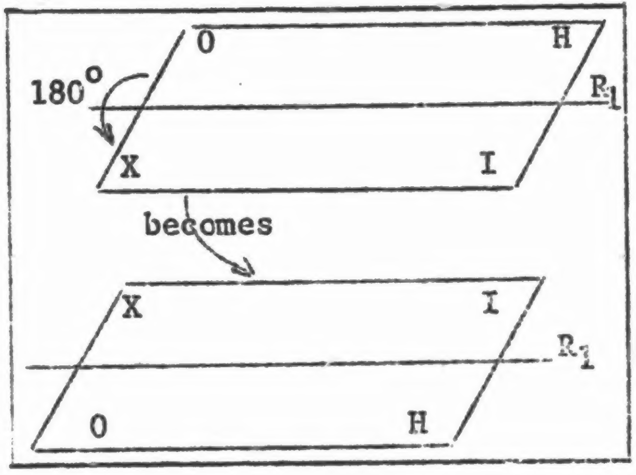
Blank Slide

RECORDED SCRIPT	WORKSHEET
(2) (B) When you grasp corners O and H and turn the rectangle over, around which axis does it turn, R_1, R_2, or R_3 ? *	1. Around which axis does the square turn? R_1, R_2, or R_3 ? ________________
(3) It turns about R_1.	
(B) Now, if I say, "Perform operation R_1," I want you to rotate the rectangle about the R_1 axis. How many degrees should the rectangle be rotated, 180° or 360°? *	2. How many degrees, 180° or 360°? ____________________

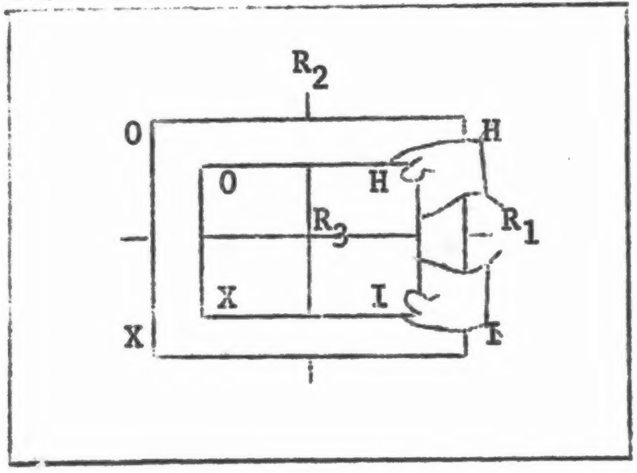
SCREEN

Left Half | Right Half

(4)


180°
O
H
R_1
X
I
becomes
X
I
R_1
O
H

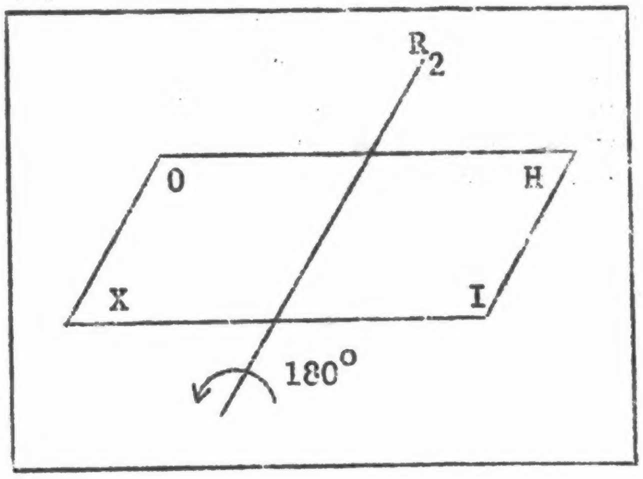
(5)


R_2
O
H
O
H
R_3
R_1
X
I
X
I

(B)

Blank Slide

(6)



RECORDED SCRIPT	WORKSHEET
(4) To perform operation R_1, rotate the rectangle about the R_1 axis 180°. Practice this once more from the beginning position. Raise your hand if you need help.	
To perform operation R_2, (5) (B) grasp corners H and I with thumb and forefinger of each hand, (6) and rotate the rectangle 180° about axis R_2.	

SYSTEM WITHOUT NUMBERS - p''' B

SCREEN

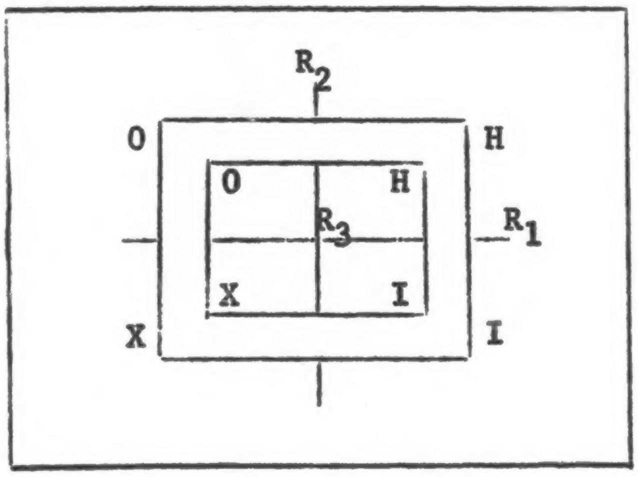
Left Half

Right Half

(M)

Operation R_2 being performed in motion pictures

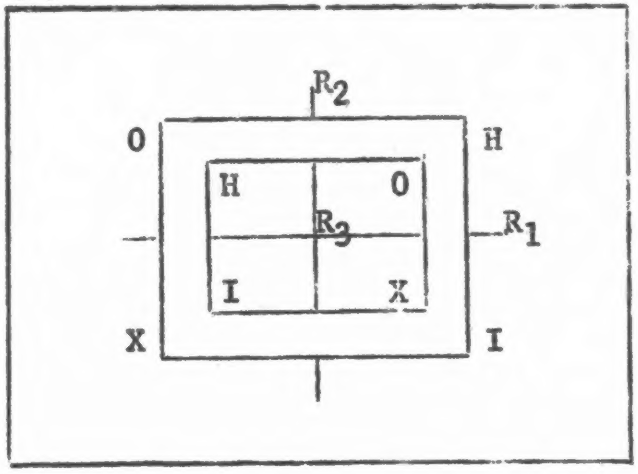
(1)


R_2
O
H
O
H
R_3
R_1
X
I
X
I

(B)

Blank Slide

(7)


R_2
O
H
H
O
R_3
R_1
I
X
X
I

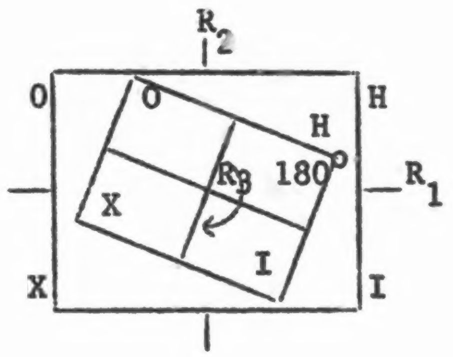
RECORDED SCRIPT

WORKSHEET

Watch this being done in motion pictures.
(M)

(1) (B) Put the rectangle in the original position. Perform operation R_2 and make sure you get the result shown. (7)

SCREEN

Left Half	Right Half
(B) Blank Slide	(B) Blank Slide
(8)  R 2 O O H H R3 180° R 1 X X I I	
(M) Operation R_3 being performed in motion pictures	

RECORDED SCRIPT **WORKSHEET**

(B) (B) To perform operation R_3, leave the rectangle lying flat on the paper. Then rotate it 180° about the R_3 axis like this. (8)

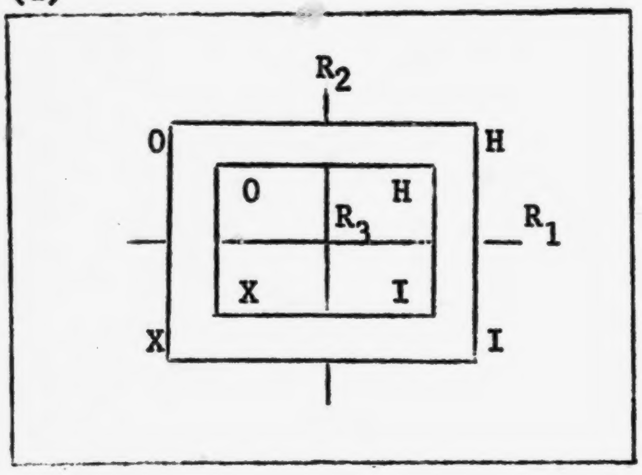
Watch operation R_3 on film.(M)(1) Did you notice that to perform operation R_3 you don't pick up the rectangle, but leave it lying flat on the paper?

SYSTEM WITHOUT NUMBERS - p''' B

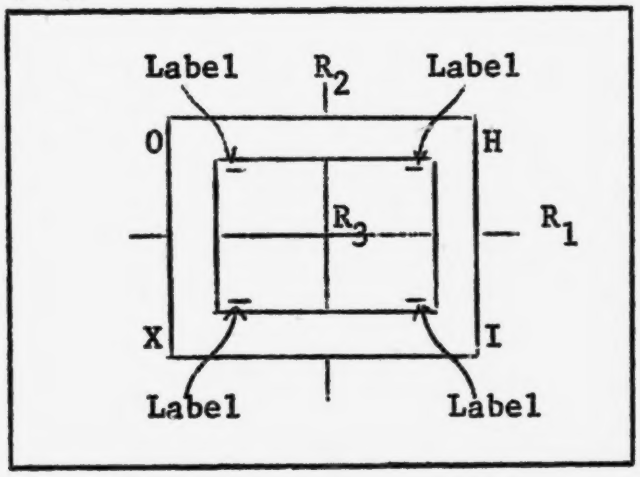
SCREEN

Left Half **Right Half**

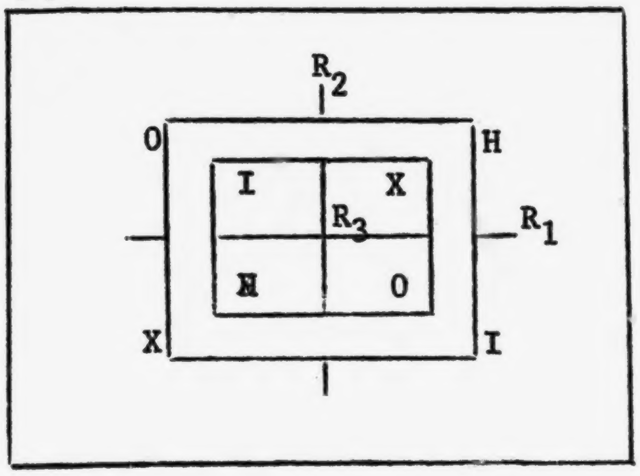
(1)


R2
O
H
O
H
R3
R1
X
I
X
I

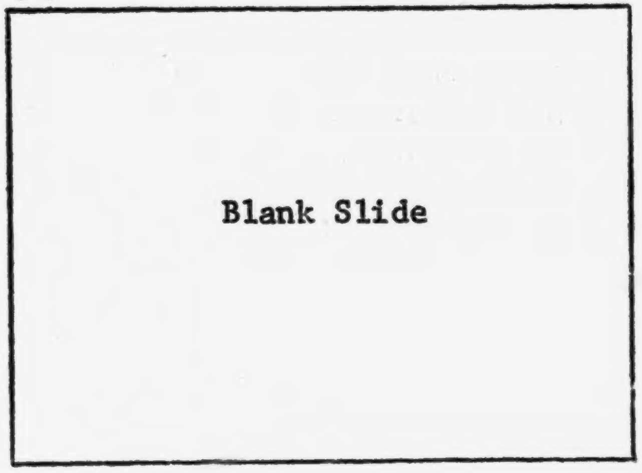
(8A)

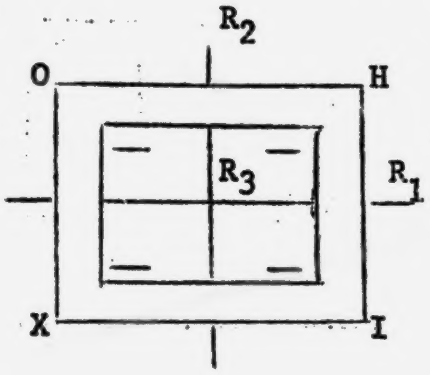

Label
R2
Label
O
H
R3
R1
X
I
Label
Label

(9)


R2
O
H
I
X
R3
R1
H
O
X
I

(B)


Blank Slide

RECORDED SCRIPT	WORKSHEET
Put the rectangle in the original position and perform operation R_3. Write what your result looks like by labeling the four corners of the rectangle in problem 3. * (8A)	3. The operation R_3: (Label the four corners of the rectangle.) 
Here is the correct result. (9) (B)	

SCREEN

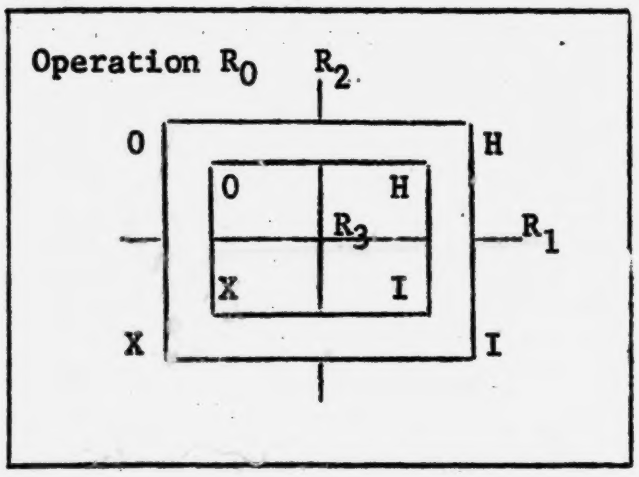
Left Half **Right Half**

(10)

R_0

No Operation

(11)


Operation R_0
R_2
O
H
O
H
R_3
R_1
X
I
X
I

(12)

Set of Operations

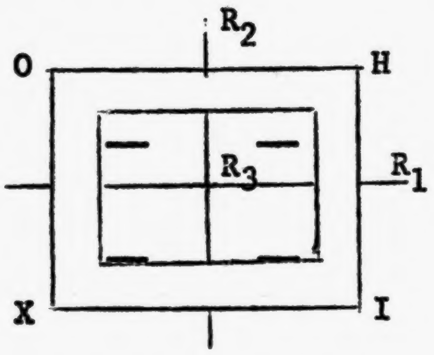
$\{R_0, R_1, R_2, R_3\}$

RECORDED SCRIPT

WORKSHEET

Return your rectangle to its original position. Let R_0 indicate no operation. (10) Now perform operation R_0 on your rectangle and write the result for item 4 on your worksheet. *

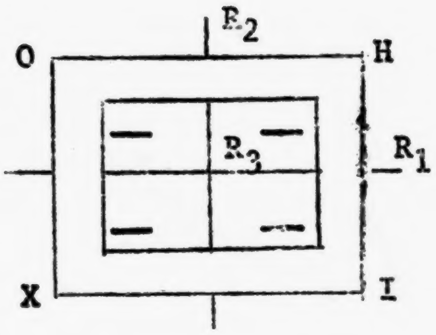
4. The operation R_0: (Label the four corners)



You should not have moved the rectangle because R_0 indicates no operation, thus your answer is this. (11)

Thus far, we have defined this set of operations. (12)
Let's practice some of these operations. Perform R_1 and write the result for problem 5. *

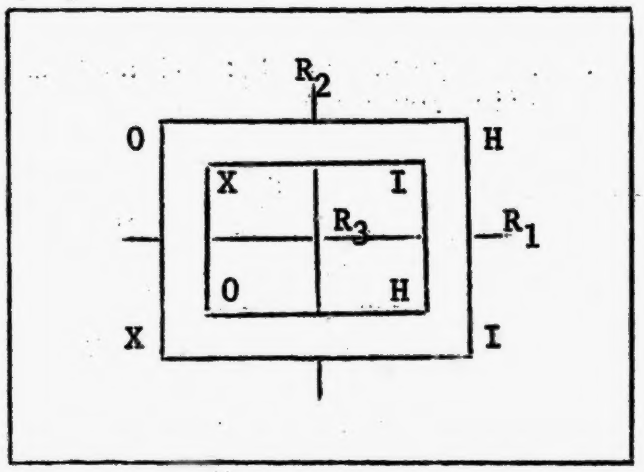
5.



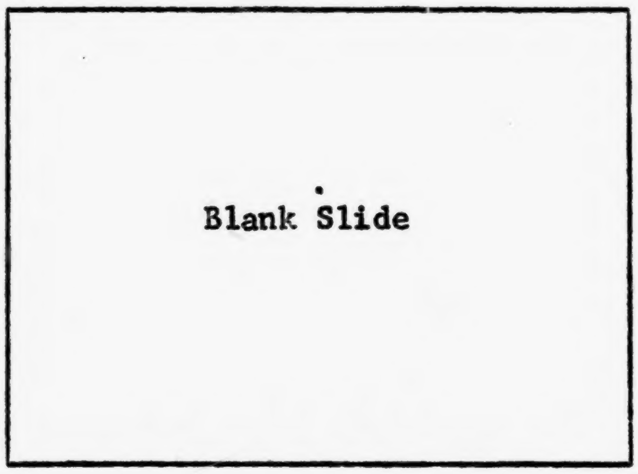
SCREEN

Left Half | Right Half

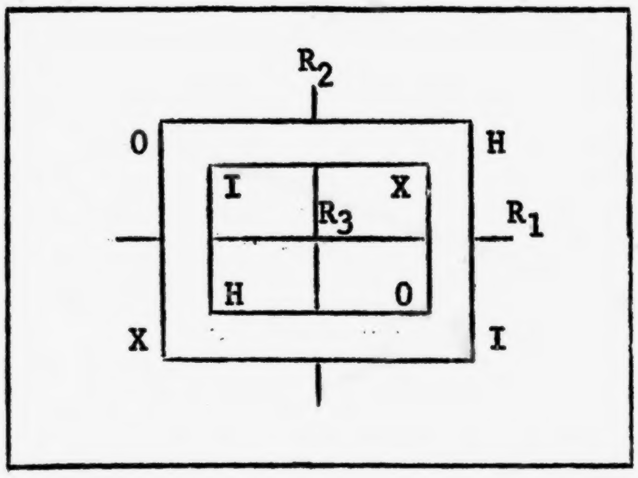
(13)


R2
O
H
X
I
R3
R1
O
H
X
I

(B)


Blank Slide

(14)

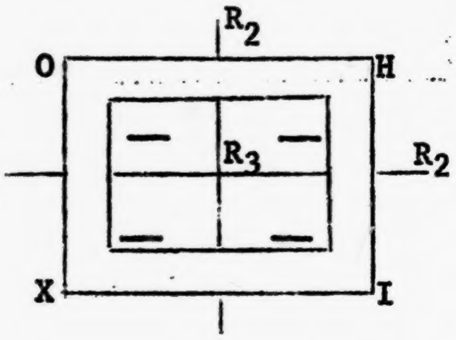

R2
O
H
I
X
R3
R1
H
O
X
I

RECORDED SCRIPT **WORKSHEET**

Here is the correct answer. (13)

(B) Be sure to start from the original position each time. Now perform R_3 and write the result. *

6.

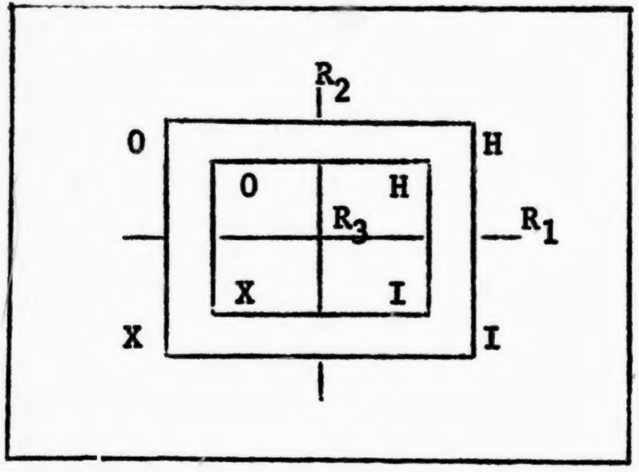


Here is R_3. (14)

SCREEN

Left Half **Right Half**

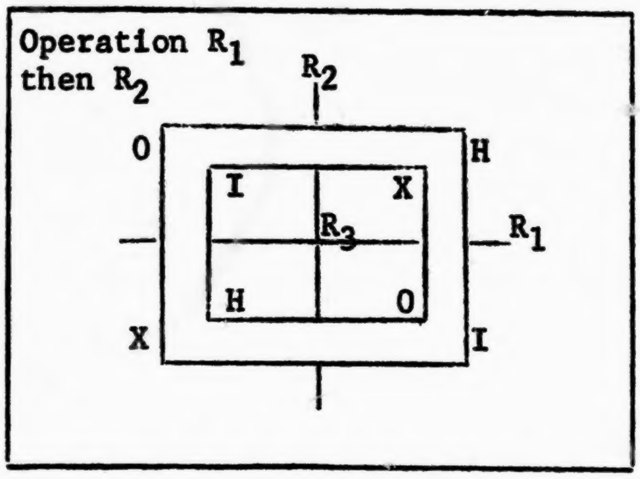
(15)


R2
O
H
O
H
R3
R1
X
I
X
I

(B)

Blank Slide

(16)


Operation R_1 then R_2
R2
O
H
I
X
R3
R1
H
O
X
I

RECORDED SCRIPT

WORKSHEET

Here is an operation. (15) Which of our set of operations is this? R_0, R_1, R_2 or R_3?*

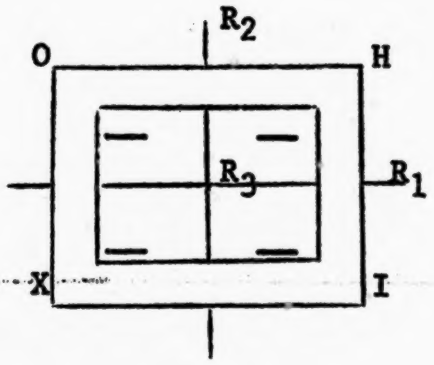
7. Which operation do you see?

____

You should have written R_0.

(B) Here is a different problem. Everyone perform operation R_1. After you have performed operation R_1, stop. Leave your rectangle in its final position. Do not go back to the original position, that is leave your rectangle in the R_1 position as it is now. With your rectangle in this position, perform operation R_2. Write your final result.*

8. Perform R_1 and then R_2.



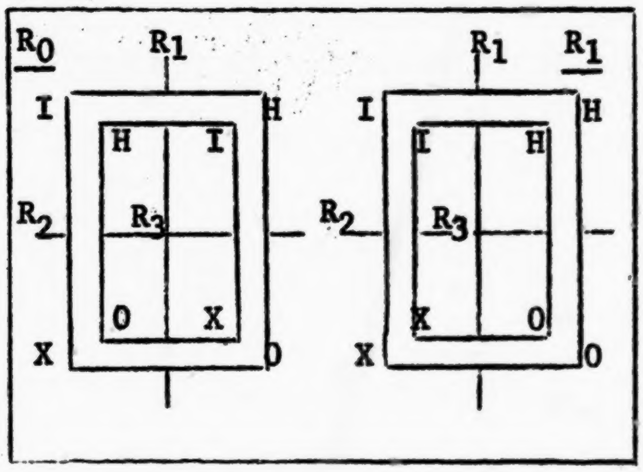
Here is the correct answer. (16)

Try R_1 and then R_2 until you get the correct answer.

SCREEN

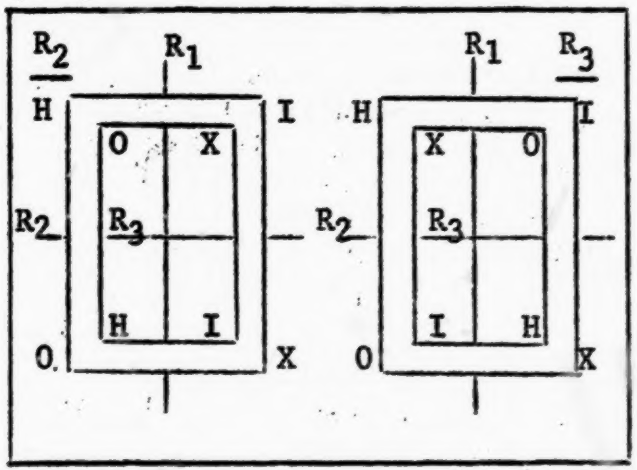
Left Half

(16A)


R0
R1
R1
R1
I
H
I
H
H
I
I
H
R2
R3
R2
R3
O
X
X
O
X
O
X
O

Right Half

(16B)


R2
R1
R1
R3
H
I
H
I
O
X
X
O
R2
R3
R2
R3
H
I
I
H
O
X
O
X

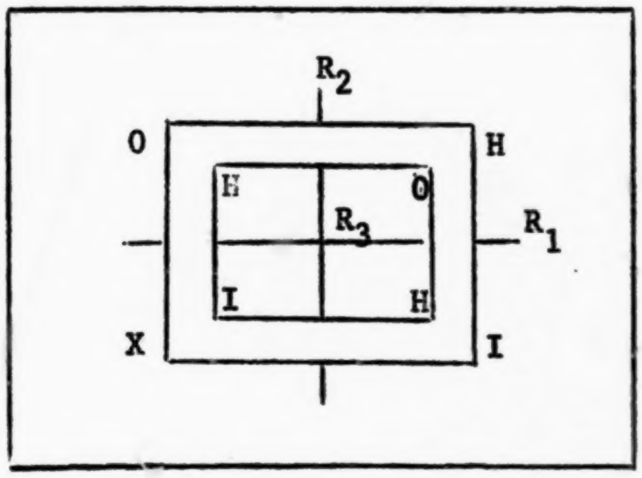
(B)

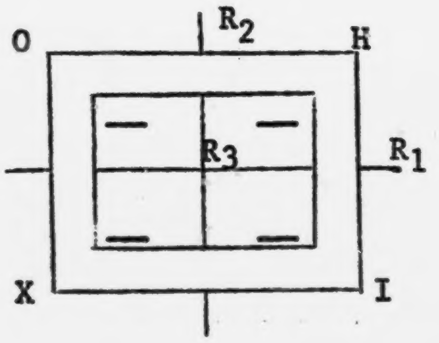
Blank Slide

(B)

Blank Slide

(17)


R2
O
H
H
O
R3
R1
I
H
X
I

RECORDED SCRIPT	WORKSHEET
When you get the correct answer, leave your rectangle in its final position as you see it on the screen. We shall assume that everyone's rectangle is in its final position as a result of performing first R_1 and then R_2. Now look at the screen. (16A) (16B) Here you see the four operations in our set. Which of these four operations matches your rectangle?*	9. The result of performing R_1 and then R_2 is identical to which operation, R_0, R_1, R_2, R_3? ________________
You should have written R_3.	
(B) (B) Try another problem. Put the rectangle in its original position. Perform R_3. Without moving back to the original position, perform R_1. Write the result in problem 10.*	10. Perform R_3 and then R_1. 
You should have this for the correct answer. (17)	

SYSTEM WITHOUT NUMBERS - p''' B

SCREEN

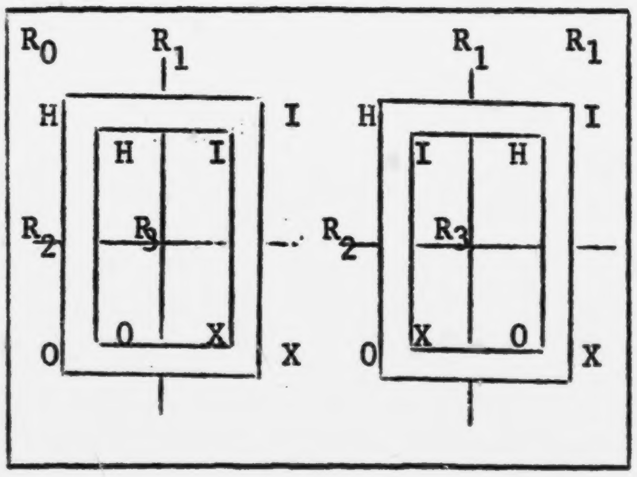
Left Half   Right Half

(18)

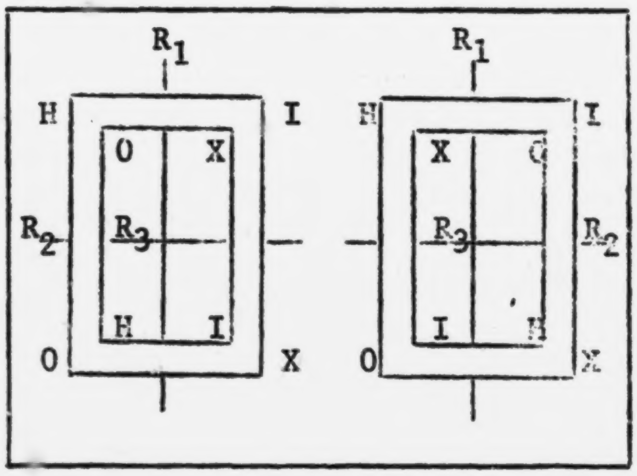
$R_3 \textcircled{f} R_1 = \underline{?}$

followed by

(16A)


R0
R1
R1
R1
H
I
H
I
H
I
I
H
R2
R3
R2
R3
0
X
X
0
0
X
0
X

(16B)


R1
R1
H
I
H
I
0
X
X
0
R2
R3
R3
R2
H
I
I
H
0
X
0
X

(B)

Blank Slide

(19)

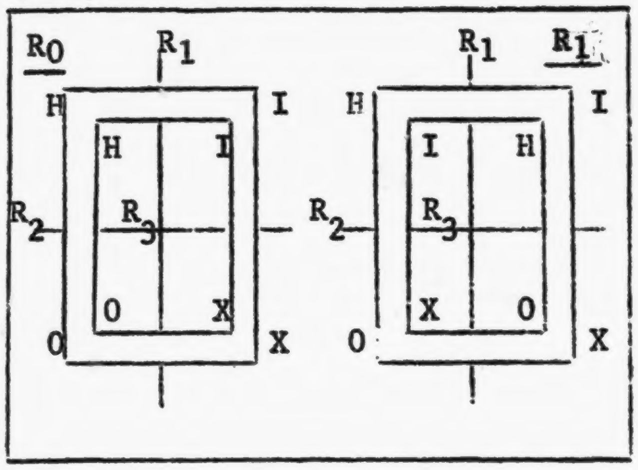
$R_3 \textcircled{f} R_1 = R_2$

RECORDED SCRIPT	WORKSHEET
See this expression. (18) The "f" with a circle around it means "followed by." The expression is read, "R_3 followed by R_1." This is what you just did.	
(16A) (16B) Compare your answer for problem 10 to what you see on the screen. Now write R_0, R_1, R_2, or R_3 for problem 11 on your worksheet.*	11. $R_3 \textcircled{f} R_1 =$ ________________
You should have written R_2. (B) (19)	

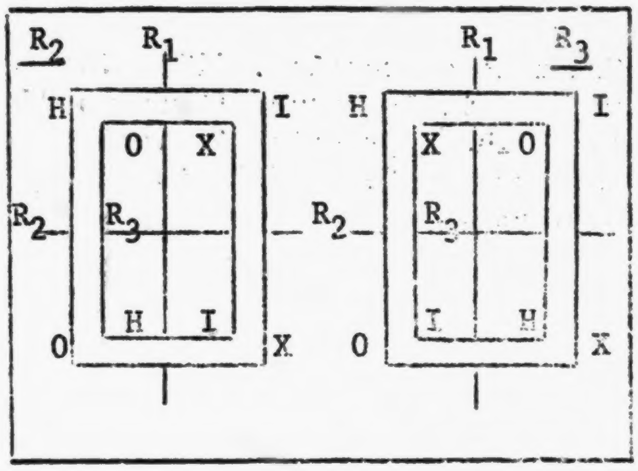
SCREEN

Left Half **Right Half**

(16A)


R_0
R_1
R_1
R_1
H
I
H
I
H
I
I
H
R_2
R_3
R_2
R_3
0
X
X
0
0
X
0
X

(16B)


R_2
R_1
R_1
R_3
H
I
H
I
0
X
X
0
R_2
R_3
R_2
R_3
H
I
I
H
0
X
0
X

(20)

10

(a) $R_0 \textcircled{f} R_1 = R_1$

(b) $R_2 \textcircled{f} R_3 = R_1$

(c) $R_3 \textcircled{f} R_2 = R_1$

(d) $R_1 \textcircled{f} R_1 = R_0$

(B)

Blank Slide

(22)

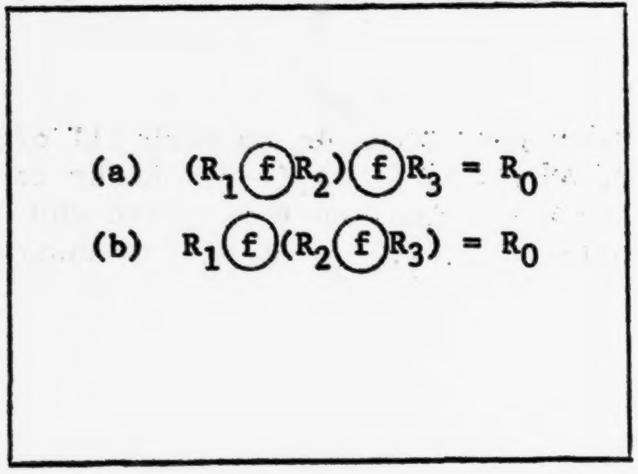
(a) $(R_1 \textcircled{f} R_2) \textcircled{f} R_3 =$

(b) $R_1 \textcircled{f} (R_2 \textcircled{f} R_3) =$

RECORDED SCRIPT	WORKSHEET
Use your rectangle to work all of Problem 12. (16A) (16B) Refer to the operations on the screen and write R_0, R_1, R_2, or R_3 for answers.*	12. (a) $R_0 \textcircled{f} R_1 =$ ____________ (b) $R_2 \textcircled{f} R_3 =$ ____________ (c) $R_3 \textcircled{f} R_2 =$ ____________ (d) $R_1 \textcircled{f} R_1 =$ ____________
Here are the correct answers. (20) (B)	
Notice problems "b" and "c". In "b", R_2 followed by R_3 results in R_1, but in problem "c", so does R_3 followed by R_2. In other words, order of operations makes no difference. What property do we call this?*	13. What property do b and c in question 12 illustrate? ____________________
This is the commutative property which we know is also a property of the integers. (22) Here are two problems, each involving two operations.	14. (a) $(R_1 \textcircled{f} R_2) \textcircled{f} R_3 =$ ________ (b) $R_1 \textcircled{f} (R_2 \textcircled{f} R_3) =$ ________
Notice that each problem is grouped differently by using parentheses. To work these problems, perform the operations in the parentheses first. Write the correct answers.*	

SCREEN

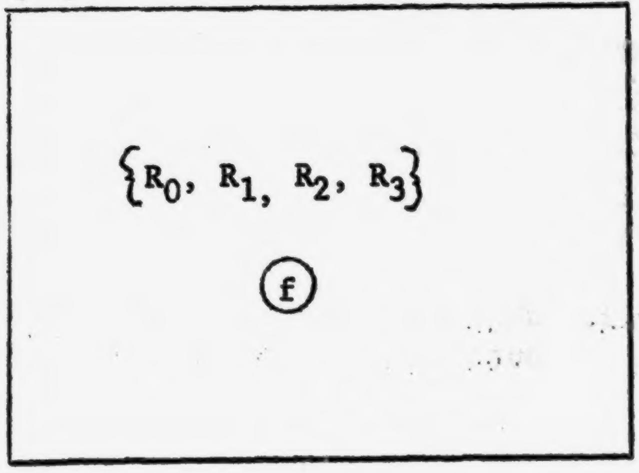
(23)



(a) $(R_1 \textcircled{f} R_2) \textcircled{f} R_3 = R_0$

(b) $R_1 \textcircled{f} (R_2 \textcircled{f} R_3) = R_0$

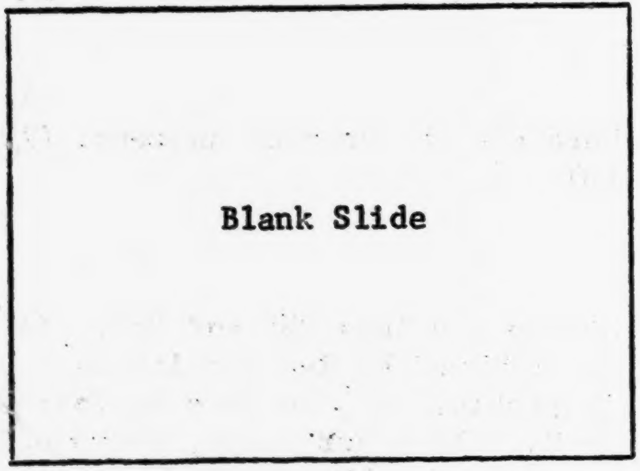
(24)



$\{R_0, R_1, R_2, R_3\}$

$\textcircled{f}$

(B)



Blank Slide

RECORDED SCRIPT	WORKSHEET
Did you get the same result for both "a" and "b"? You should have. (23)	
What property do these problems illustrate?*	15. What property does (14) illustrate? ________________
This is an example of associative law.	
(24) (B) So our set of four elements - R_0, R_1, R_2, and R_3 - and the operation "followed by" have the commutative and associative properties just as our familiar system of integers has. We could investigate other properties which we know integers have to see if they are properties of this system also.	

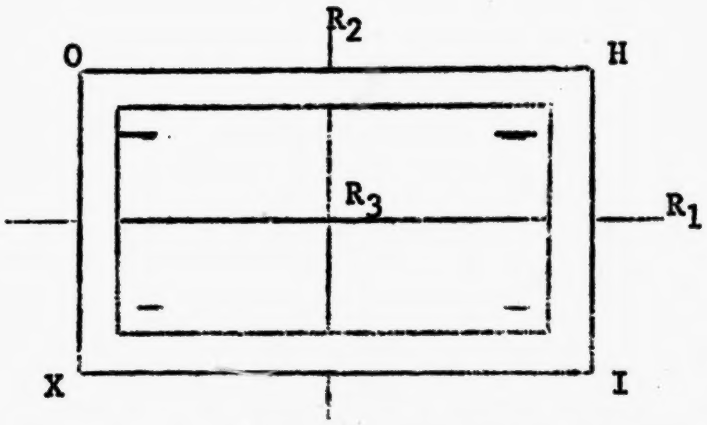
SYSTEM WITHOUT NUMBERS:
MEASURE Y_3

Use the diagram and your rectangle whenever you need to.

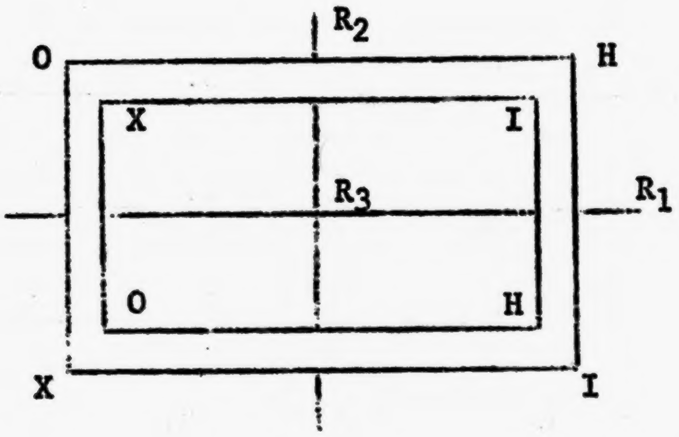
ANSWER:

1. Write the results of performing operation R_3. (Be sure to start in the original position.)

	I	X
1.	H	O



2. Starting from the original position, one of the four operations was performed.



(a) About which axis was the rectangle rotated?

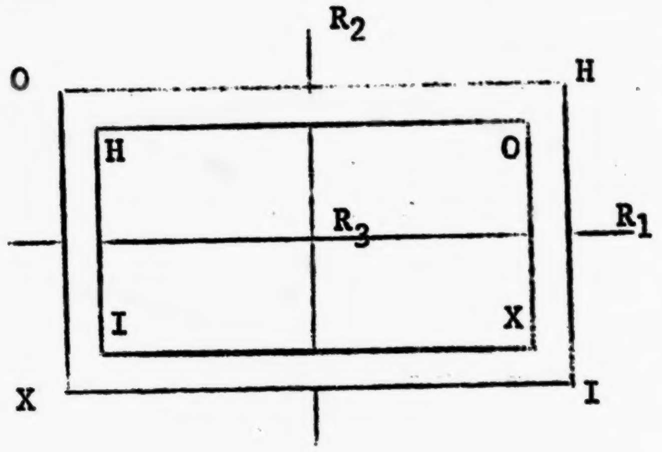
(b) Which single operation was performed?

2. a.) R_1

b.) R_1

ANSWER:

3. Starting from the original position two operations were performed. The second operation followed the first without first going back to the original position.



What two operations were performed?

(a) First operation ______ was performed.

(b) Second, operation ______ was performed.

3.

a)	ⓕ	b)
R_1		R_3
R_2		R_0
R_0		R_2
R_3		R_1

4. When you perform operation R_0, how many degrees does the rectangle rotate?

4. 0

5. R_2 ⓕ R_2 = ______________

5. R_0

6. R_2 ⓕ R_3 = ______________

6. R_1

7. R_1 ⓕ R_0 ⓕ R_3 ⓕ R_1 = ________

7. R_3

8. (R_3 ⓕ R_0) ⓕ R_1 = (R_0 ⓕ R_3) ⓕ R_1

What property does the above illustrate:

(a) commutative

(b) distributive

(c) associative

(d) It doesn't illustrate any of the above properties.

8. c

	ANSWER:
9. (a) Complete the following: $R_3 \textcircled{f} R_2 = R_3 \textcircled{f}$ ________	9. a) R_2
(b) What property does (a) illustrate: (a) commutative (b) distributive (c) associative (d) It doesn't illustrate any of the above properties.	b) d
10. Write an equation to illustrate an identity element if you think one exists.	10. $R_0 \textcircled{f} R_0 = R_0$, or $R_1 \textcircled{f} R_0 = R_1$, or $R_2 \textcircled{f} R_0 = R_2$, or $R_3 \textcircled{f} R_0 = R_3$

Point Distribution: one point each.

Total Points: ten.

GRAPHING OF INEQUALITIES

Introduction to the Graphing of Inequalities Program

As discussed in Chapter 3 of Part I, the impracticality of incorporating the experiment directly into the instructional sequence, necessitated the development of additional programs. The intern on the math team wrote Graphing of Inequalities, which is presented here in its entirety: three versions of both A and B. It is presented completely because it illustrates a major revision, as well as some of the problems a beginning programmer is likely to encounter.

The faults of the original p'A and p'B versions were mainly two. First, they assumed too much with regard to the students' background; the students simply did not have the facility in graphing of straight lines that was anticipated. Secondly, the program tried to teach too much at one time; this resulted not only in the presentation of more ideas than the student could grasp, but also in a program that was longer than desirable for purposes of the experiment.

Versions p'A and p'B review briefly the graphing of an equation which is then related to the graphing of inequalities. Several forms of inequalities are considered next beginning with $x \geq y$, progressing to $ax + by \leq c$ and making the distinction between $ax + by \leq c$ and $ax + by < c$. In addition, forms like $ax \leq c$ are considered.

The first revisions, p'', of both A and B takes just the first of these concepts, the graphing of an equation, and reviews this idea in three steps: finding points which satisfy a given equation, plotting these points, and finding the straight line through the plotted points. In p''B, equations which graph as lines parallel to the x- or y-axes are also considered, while in p''A, no special mention is made of such equations.

The example $x = -3$ was included in p'' A as this was a troublesome point for the high ability students. The main revision in p'''B was to consider the detailed steps in finding points which satisfied an equation of the form $ax + by = c$; a review using a slide pointing out the steps necessary to graph an equation was included at the end of p''' B.

Although a set of co-ordinate axes is not always indicated in the reproduction of the workbooklet that follows, such a grid was included in the student's workbooklet when necessary. The same holds true for the measures Y_1, Y_2, Y_3, which follow the programs. The major change in the program from p' to p'' necessitated a corresponding change in measures Y_1 and Y_2. In the other programs, measures Y_1, Y_2, Y_3 are much more similar, as are Y_2 and Y_3 for this program.

GRAPHING OF INEQUALITIES:

ORIGINAL PROGRAM p' A

GRAPHING OF INEQUALITIES - p' A

SCREEN

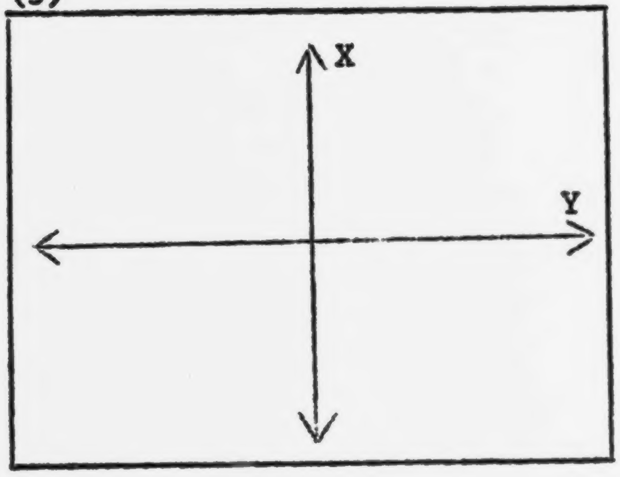
Left Half

Right Half

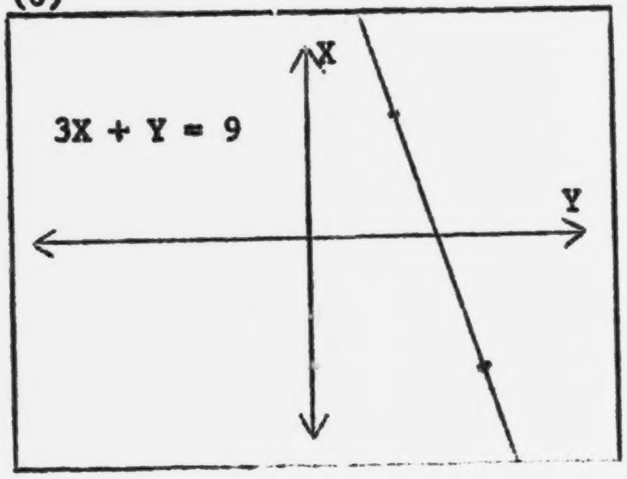
(2)

3X + Y = 9

(3)



(6)



(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
You have worked with equations like $3x + y = 9$, (2) and sketched their graphs in the rational number plane. (3) You did this by plotting two points which satisfied the equation and drawing the straight line joining them, for the graph of a linear equation is a straight line. (6) So any point on our straight line should make $3x + y = 9$ a true statement. Pick a point on the line and try it.*	1. Pick a point on the line $3x + y = 9$ (,). Check the point to see if it satisfies the equation: ______________________ ______________________ ______________________ ______________________
(B) But an equation is only one kind of number statement. For instance, you have graphed $x \geq 3$ on the number line.	

GRAPHING OF INEQUALITIES p' A

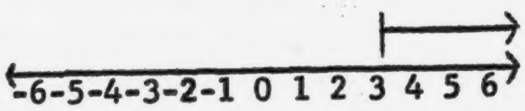
SCREEN

Left Half

Right Half

(9)

$X \geq 3$


-6-5-4-3-2-1 0 1 2 3 4 5 6

(10)

$3X + Y \leq 9$

(10A)

$X \geq Y$

(11)

$X = Y$

RECORDED SCRIPT

WORKSHEET

(9) Why couldn't we graph (10) a number statement like $3x + y \leq 9$?

Such a statement is called an inequality. The graphing of inequalities has important applications for industry in linear programming, a topic we will look at later. Right now, let's find out how we might graph an inequality, for instance $x \geq y$. (10A)

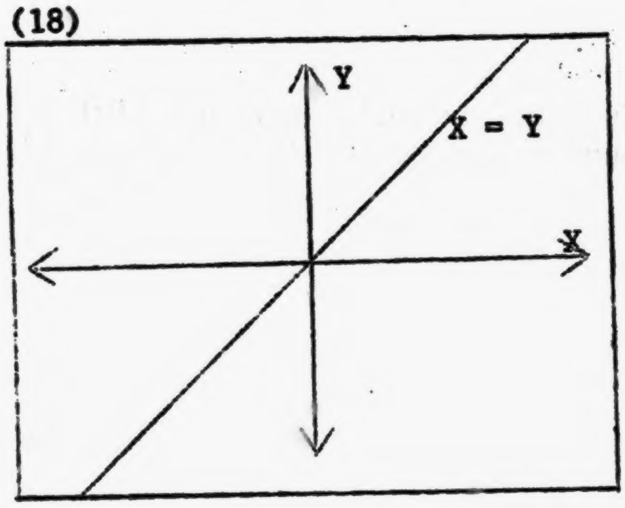
First consider the simpler equation $x = y$. (11) Graph this equation.*

2. Graph $x = y$

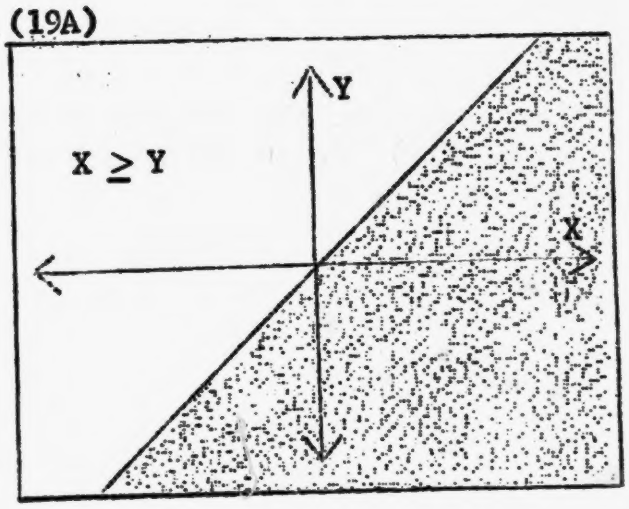
SCREEN

Left Half **Right Half**

(18)



(19A)



RECORDED SCRIPT	WORKSHEET
The graph is this straight line (18) passing through the origin.	
Where do you think the graph of $x > y$ lies in relation to the graph of $x = y$? On your graph of $x = y$ in question 2, shade the area you think represents the graph of $x > y$. Check your result by picking a point in your shaded area. Does it satisfy $x > y$?*	3. Shade $x > y$ in question 2. Pick a point in the area you shaded. (,) Does it satisfy $x > y$? (Yes or No) __________
You should have concluded that the graph of $x \geq y$ looks like this. (19A)	

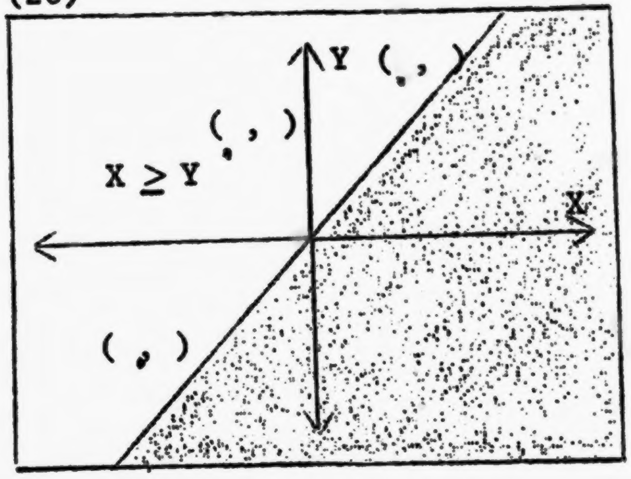
GRAPHING OF INEQUALITIES - p' A

SCREEN

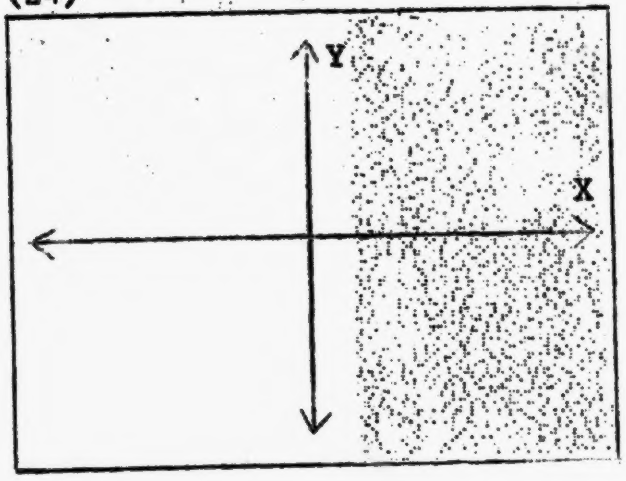
Left Half

Right Half

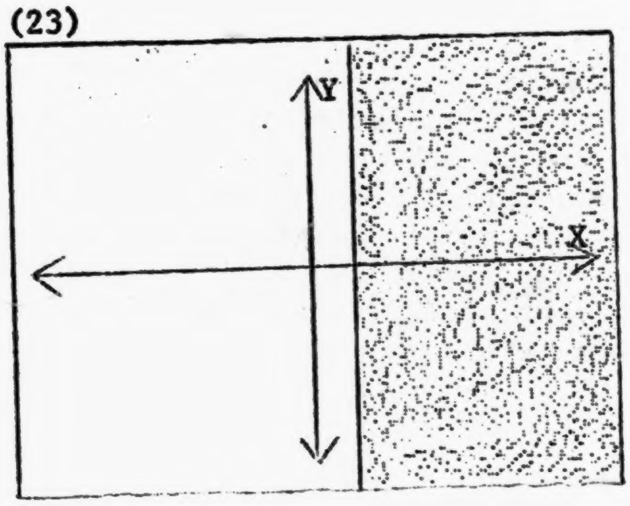
(20)

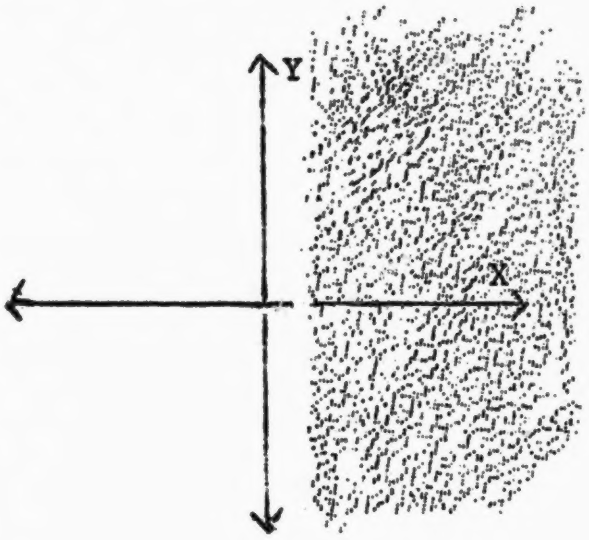
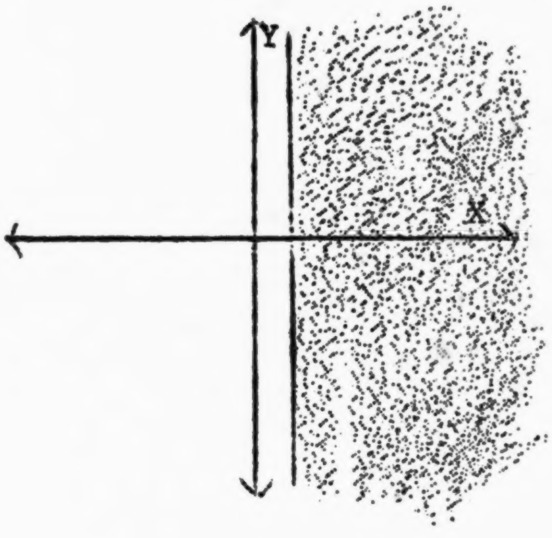


(24)



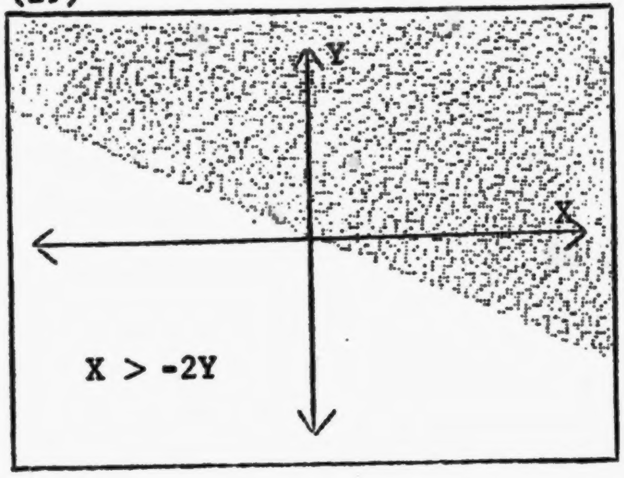
(23)

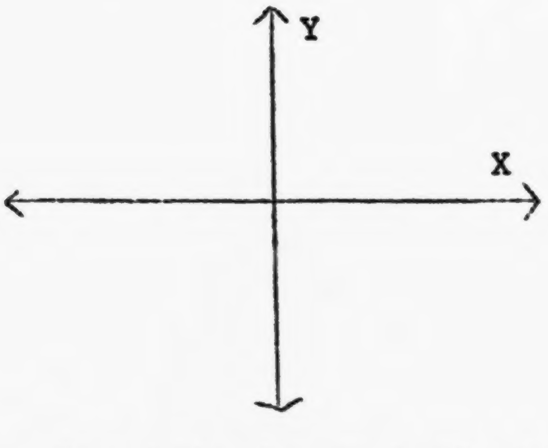


RECORDED SCRIPT	WORKSHEET
Is it possible that the points indicated here (20) in blue are also a part of the graph of $x \geq y$? Check one of the points to see if it satisfies $x \geq y$. *	4. Pick one of the points on the screen. (,) Does it satisfy $x \geq y$? (Yes or No) ____________
(24) (23) Here you see two graphs which differ slightly. See if you can write down a number statement (equation or inequality) which represents each. * The graph on the left is the graph of $x > 1$, the graph on the right is the graph of $x \geq 1$. The difference is the straight line $x = 1$	5. equation: ____________________  equation: ____________________ 

SCREEN

Left Half	Right Half
(26) X > -2Y	(B) Blank Slide
	(27) X = -2Y
	(29)

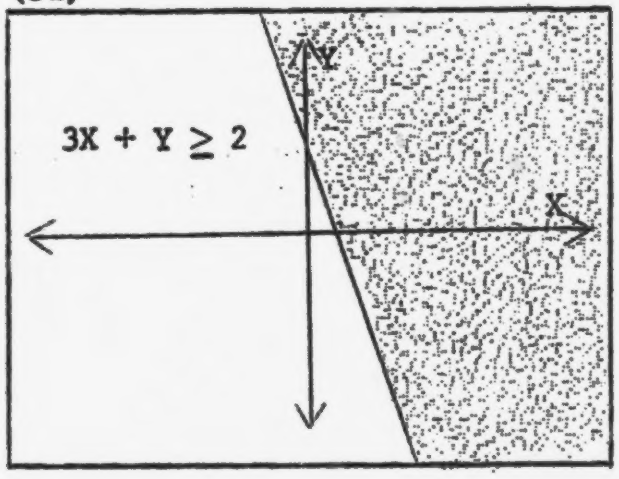


RECORDED SCRIPT	WORKSHEET
If we want to graph the inequality (26) (B) $x > -2y$, it is easier to first graph an equation. What equation would be helpful in graphing $x > -2y$?*	6. What equation is related to the inequality $x > -2y$? ______________________________
(27) $x = -2y$ would be a good equation to consider. Graph this equation and then shade the graph of $x > -2y$.* Is the line $x = -2y$ a part of the graph of $x > -2y$?	7. Graph $x = -2y$  Now shade the graph of $x > -2y$.
(29) The correct graph of $x > -2y$ does not include the line $x = 2y$.	8. Is the line $x = -2y$ a part of the graph of $x > -2y$? Explain.

SCREEN

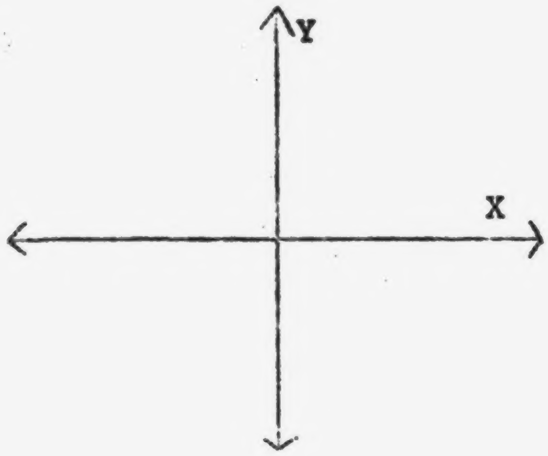
Left Half	Right Half
(30) $3X + Y \geq 2$	(B) Blank Slide

(31)



Left Half	Right Half
(B) Blank Slide	(B) Blank Slide

RECORDED SCRIPT	WORKSHEET
Now see if you can graph $3x + y \geq 2$.* (30) (B)	9. Graph $3x + y \geq 2$



Here is the correct graph. (31) Note that since the equation is $3x + y > 2$ or $3x + y = 2$, the line $3x + y = 2$ is included as part of the graph.

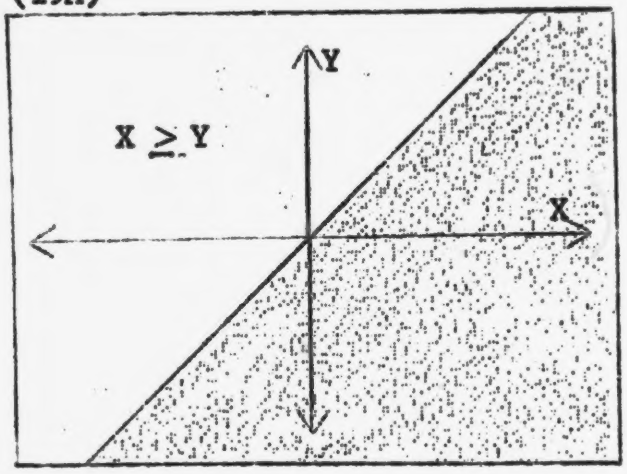
(B) (B) Now that you have an idea of how to graph inequalities, consider this problem.

SCREEN

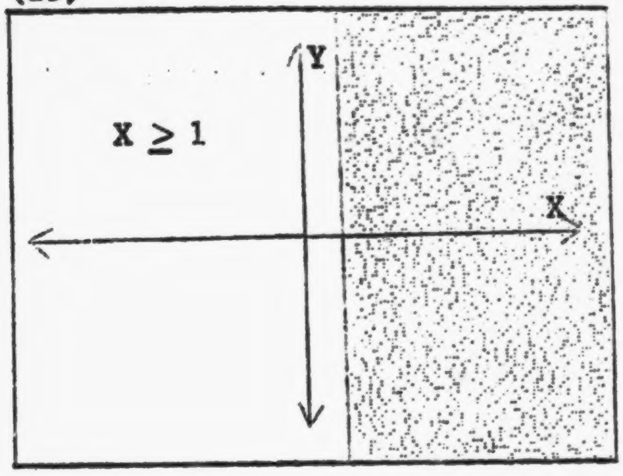
Left Half

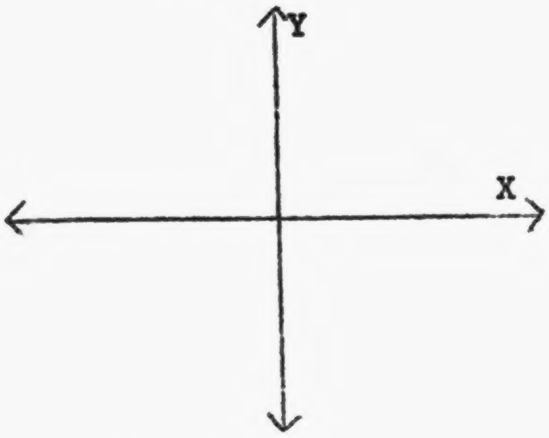
Right Half

(19A)



(25)



RECORDED SCRIPT	WORKSHEET
What if we wanted the graph of all the points (19A) (25) which satisfied both $x \geq y$ and $x \geq 1$? Graph both these inequalities on the same number plane, shading each a different color. How will the points which satisfy both inequalities be shaded?*	10. Graph $x \geq y$ and $x \geq 1$ on the same number plane. (Use ink for one, pencil for the other.) 
We will explore this kind of problem further and see how this idea can be used to solve some interesting problems.	

GRAPHING OF INEQUALITIES:

FIRST REVISION p'' A

SCREEN

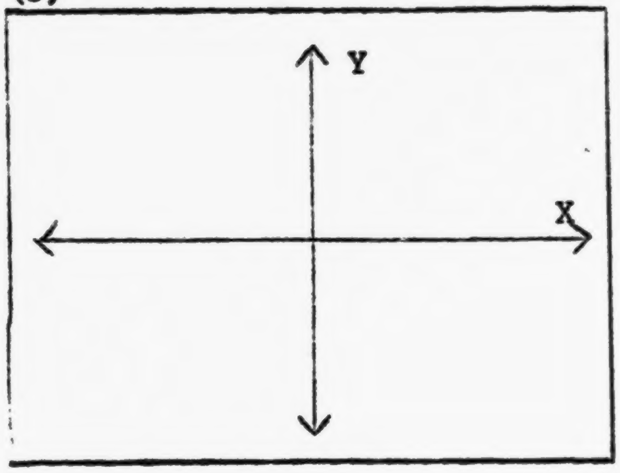
Left Half

(2)

$3X + Y = 9$

Right Half

(3)



(B)

Blank Slide

(8)

(3, 0)

$3 \cdot 3 + 0 = 9$

$\therefore 9 = 9$

RECORDED SCRIPT

WORKSHEET

In order to have a picture of an equation (2) such as $3x + y = 9$, we often graph such an equation in the rational number plane. (3) What does it mean to graph an equation, and how do we go about doing it?

(B) First, what do we mean when we say that the number pair (3,0) satisfies the equation $3x + y = 9$? We mean that if we substitute 3 for x and 0 for y in $3x + y = 9$, we get a true statement. (8)

SCREEN

Left Half	Right Half
(36) DOES (-2,1) SATISFY $3X + Y = 9$?	(B) Blank Slide
	(37) $3X + Y \overset{?}{=} 9$ $3 \cdot 2 + 1 \overset{?}{=} 9$ $-5 \neq 9$
(4R) (2,3) (4,-3)	(B) Blank Slide

RECORDED SCRIPT	WORKSHEET
(36) (B) Does the point (-2,1) satisfy $3x + y = 9$?*	1. Does (-2,1) satisfy $3x + y = 9$? ________________ ________________ ________________ Yes or No ________
No, because (37) when we substitute it in, we get the result $-5 = 9$, certainly not a true statement.	
(4R) (B) Check the points (2,3) and (4,-3) to see if they satisfy $3x + y = 9$.*	2. Do (2,3), (4,-3) satisfy $3x + y = 9$? ________ ________ ________ ________ ________ ________ (2,3) Yes or No _____ (4,-3) Yes or No _____

SCREEN

Left Half

Right Half

(4)

(2,3)
$3 \cdot 2 + 3 = 9$

(4,-3)
$3 \cdot 4 + -3 = 9$

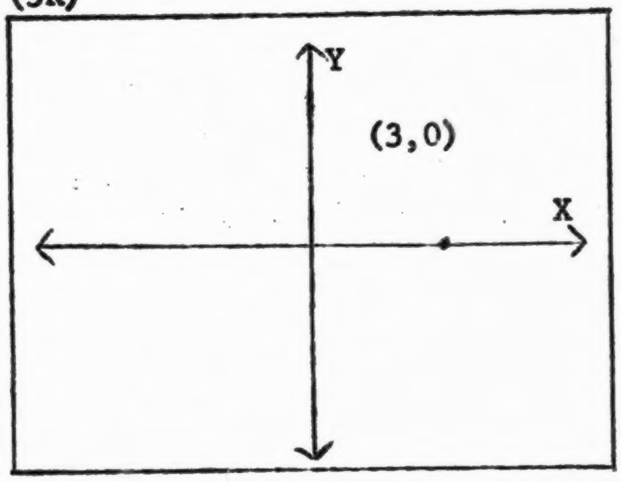
(8R)

(3, 0)

(B)

Blank Slide

(3R)



RECORDED SCRIPT

WORKSHEET

Here are (4) the results of substituting the points in the equation. Note that in both cases we get a true statement.

Secondly, (8R) (B) let's take a look at a number pair, for instance (3,0). The first coordinate, in yellow, also called the x-coordinate, is three. The second or y-coordinate, in green, is 0. So to plot (3R) this ordered pair on the number plane, go three units in the x-direction and zero units in the y-direction.

SCREEN

Left Half

Right Half

(4R)

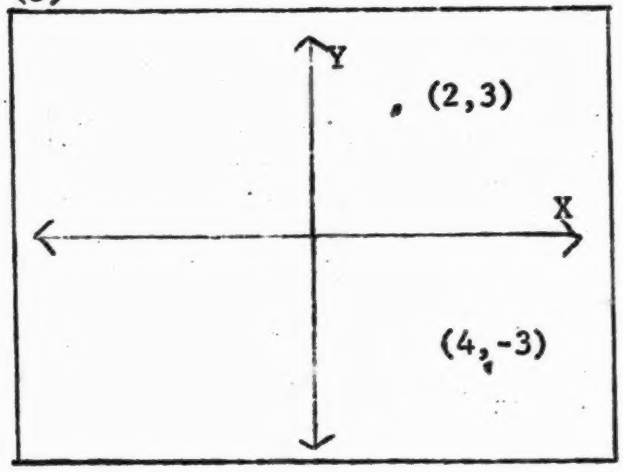
(2,3)

(4,-3)

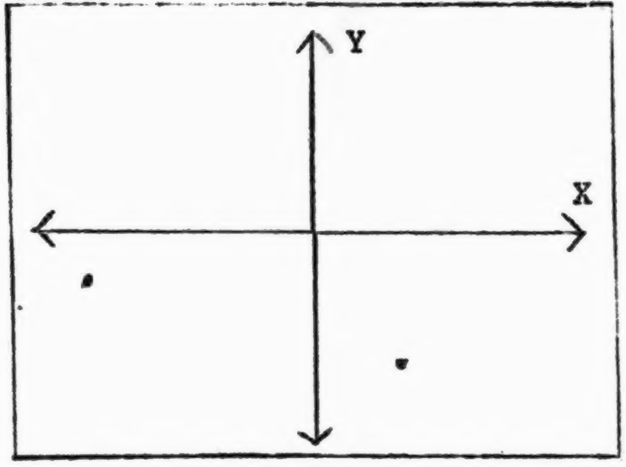
(B)

Blank Slide

(5)

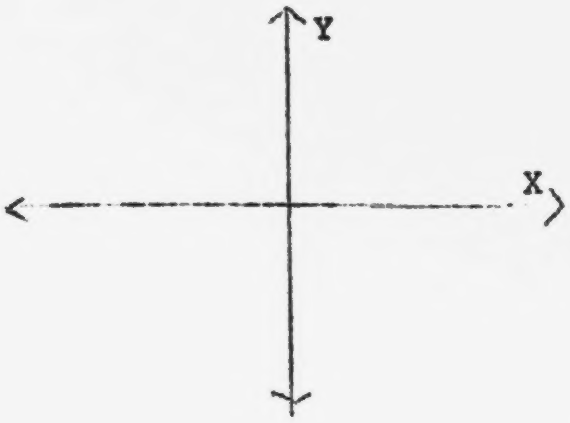


(3RR)



(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
Try plotting these number pairs, (4R) (B) (2,3) and (4,-3).*	3. Plot (2,3) and (4,-3). 
(5) The plotted pairs look like this.	
Can we also write down the ordered pair that corresponds to a given point in the plane? On the screen (3RR) (B) are two such points. See if you can write down an ordered pair which represents each.*	4. The two points are: (,) (,).

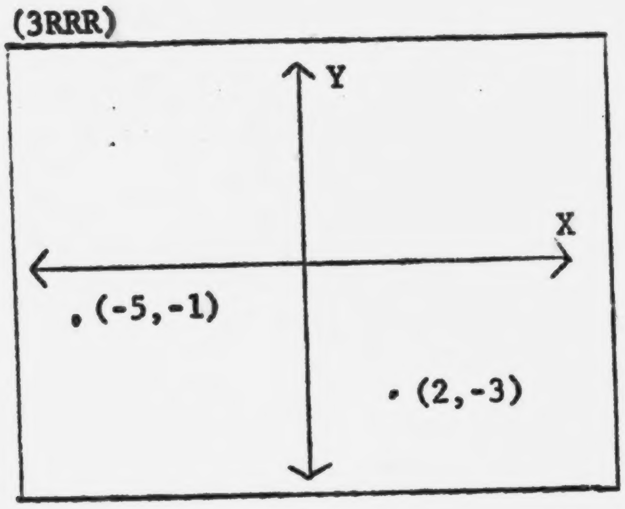
GRAPHING OF INEQUALITIES - p'' A

SCREEN

Left Half

Right Half

(3RRR)



(B)

Blank Slide

(B)

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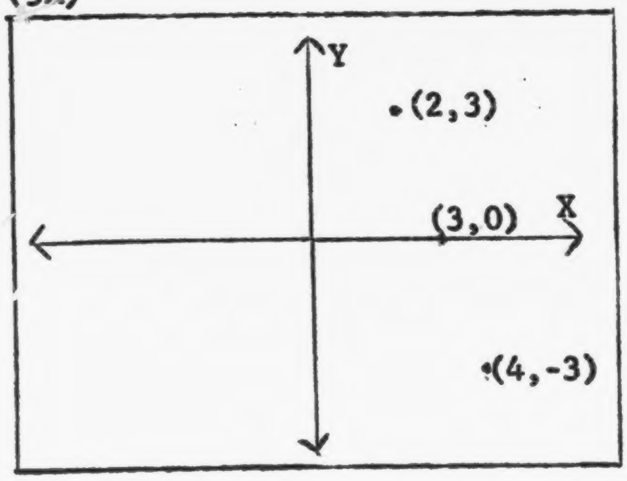
(5R)



RECORDED SCRIPT

WORKSHEET

The point (3RRR) on the left is (-5,-1), the one on the right (2,-3).

(B) (B) Thirdly, equations of the form 3x + y = 9, x = 4, x-5y = 2 are called linear equations, because their graphs are straight lines.

We have already found that these three number pairs, (2,3), (4,-3) and (3,0) satisfy 3x + y = 9 (5R) and we plotted these points.

SCREEN

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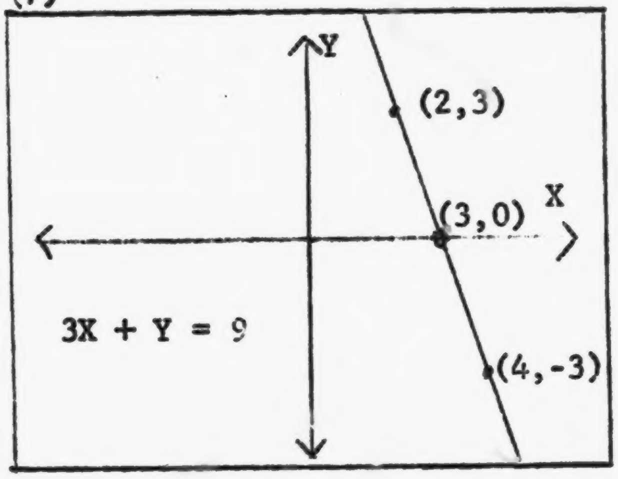
(7)


Y
(2,3)
X
(3,0)
3X + Y = 9
(4,-3)

(27)

X = -2Y

(B)

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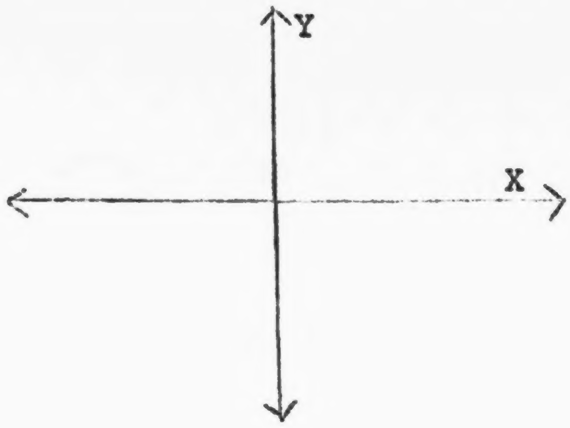
RECORDED SCRIPT	WORKSHEET
So, to draw the graph of $3x + y = 9$, (7) we simply draw the straight line between these points. Note that the line pasees through all three points.	
(27) (B) Let's say you are given the equation $x = -2y$ and you want to draw its graph. First, how will you find points that satisfy $x = -2y$? You could pick a value for y, say $y = 2$. What is x then?* $X = -4$. What is x if $y = 0$? If y is -1 1/2?* If y is 0, x is 0. If y is -1 1/2, x is 3.	5. If $y = 2$, what is x? (,2) 6. $y = 0$, what is x? (, 0) $y = -1\ 1/2$, what is x? (, - 1 1/2
Second, plot these points, and third, draw the straight line joining them to get the graph of $x = -2y$.*	7. Plot the points in 5 and 6 and draw the graph of $x = -2y$. 

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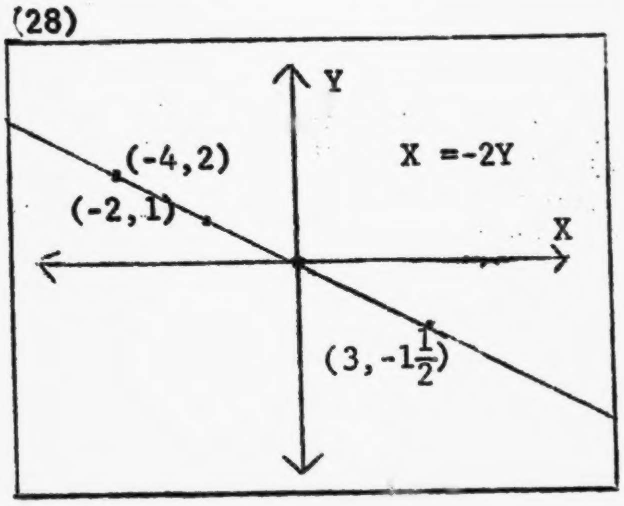
(28)


Y
(-4,2)
X =-2Y
(-2,1)
X
(3,-1$\frac{1}{2}$)

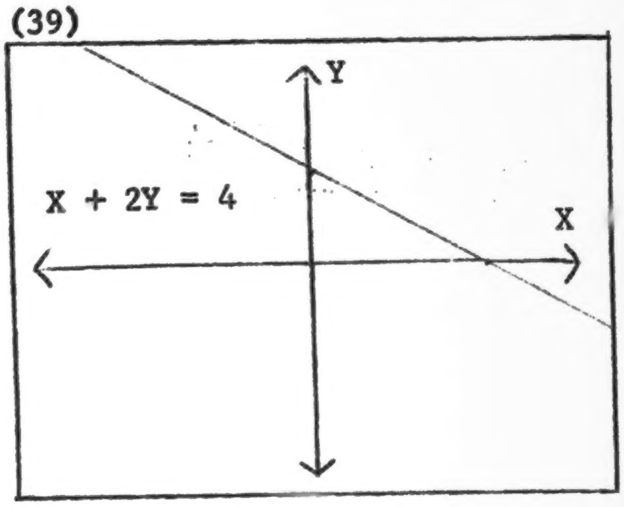
(38R)

X + 2Y = 4

(B)

Blank Slide

(39)



RECORDED SCRIPTS

WORKSHEET

$x = -2y$ looks like this. (28)

(38R) (B) Now try graphing $x + 2y = 4$. If you are stuck as to how to begin, solve the equation for x and find three number pairs which satisfy the equation. Then plot them and draw the line.*

Here is the result. (39) Note that the third point serves to check our work.

If the line does not go through all three points, we can go back and check our computations and the plotting of our number pairs.

GRAPHING OF INEQUALITIES:

SECOND REVISION p'''A

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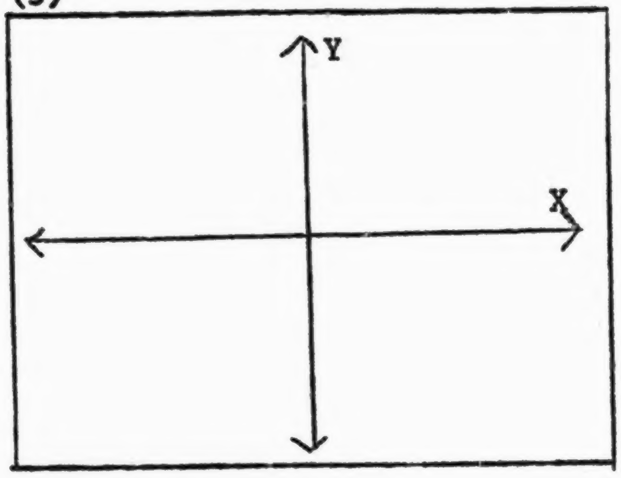
(2)

$3X + Y = 9$

(3)



(B)

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(8)

(3, 0)

$3 \cdot 3 + 0 = 9$

$\therefore 9 = 9$

RECORDED SCRIPT

WORKSHEET

In order to have a picture of an equation (2) such as 3x + y = 9, we often graph such an equation in the rational number plane. (3) What does it mean to graph an equation, and how do we go about doing it?

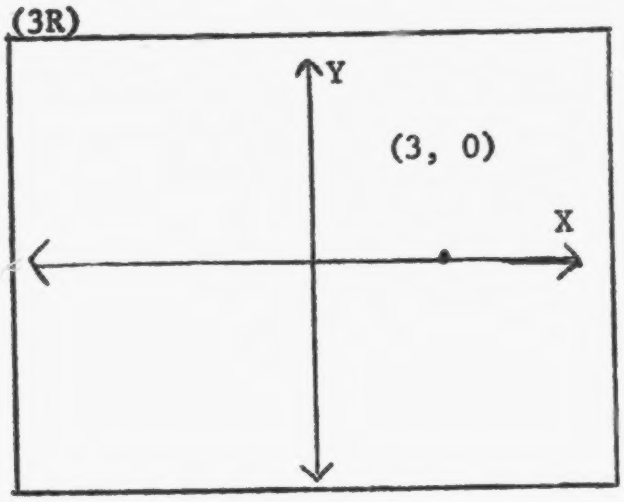
(B) First, what do we mean when we say that the number pair (3,0) satisfies the equation (8) 3x + y = 9? We mean that if we substitute 3 for x and 0 for y in 3x + y = 9, we get a true statement.

SCREEN

Left Half	Right Half
(36) DOES (-2,1) SATISFY $3X + Y = 9$?	**(B)** Blank Slide
	(37) $3X + Y \overset{?}{=} 9$ $3 \cdot {}^{-}2 + 1 \overset{?}{=} 9$ $-5 \neq 9$
(4) (2,3) $3 \cdot 2 + 3 = 9$ (4,-3) $3 \cdot 4 + -3 = 9$	**(B)** Blank Slide

RECORDED SCRIPT	WORKSHEET
(36) (B) Does the point (-2,1) satisfy $3x + y = 9$?*	1. Does (-2,1) satisfy $3x + y = 9$? ____________________ ____________________ ____________________ Yes or No __________
No, because (37) when we substitute it in, we get the result $-5 = 9$, certainly not a true statement.	
(4) (B) Note here that the points (2,3) and (4,-3) satisfy $3x + y = 9$, because when we substitute them in the equation, we get true statements.	

SCREEN

Left Half	Right Half
(8R) (3, 0)	(B) Blank Slide
	(3R) 
(4R) (2, 3) (4, -3)	(B) Blank Slide

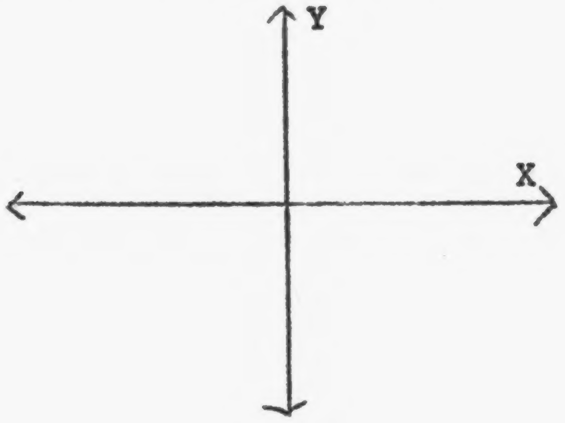
RECORDED SCRIPT

WORKSHEET

Secondly, (8R) (B) let's take a look at a number pair, for instance (3,0). The first coordinate, in yellow, also called the x-coordinate, is three. The second, or y-coordinate, in green, is 0. So to plot (3R) this ordered pair on the number plane, go three units in the x-direction and zero units in the y-direction.

Try plotting these number pairs, (4R) (B) (2,3) and (4,-3).*

2. Plot (2,3) and (4,-3).



SCREEN

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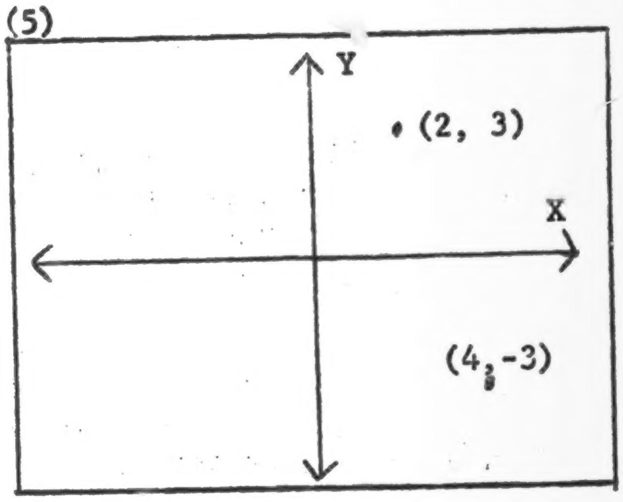
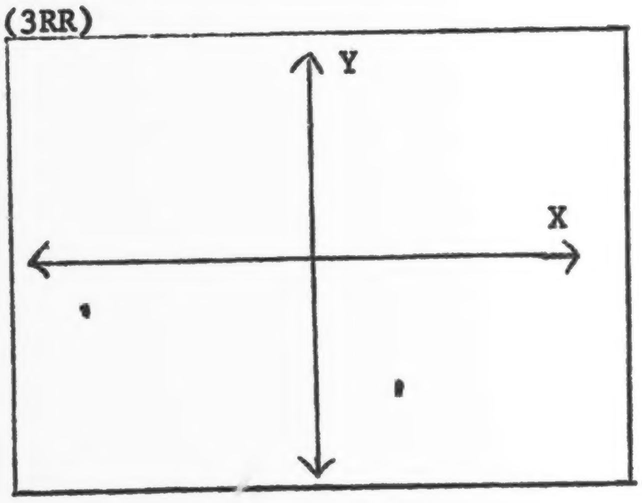
(5)



(3RR)



(B)

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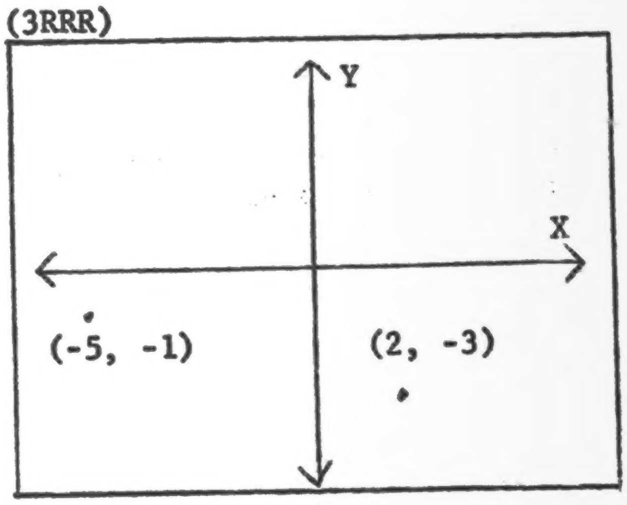
(3RRR)



RECORDED SCRIPTS	WORKSHEET
(5) The plotted pairs look like this.	
Can we also write down the ordered pair that corresponds to a given point in the plane? On the screen (3RR) (B) are two such points. See if you can write down an ordered pair which represents each.*	3. The two points are: (,) (,).
The point (3RRR) on the left is (-5,-1), the one on the right (2,-3).	

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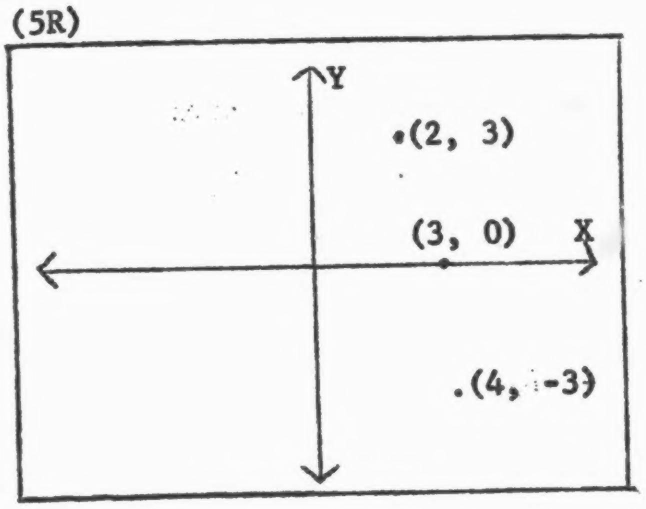
(B)

Blank Slide

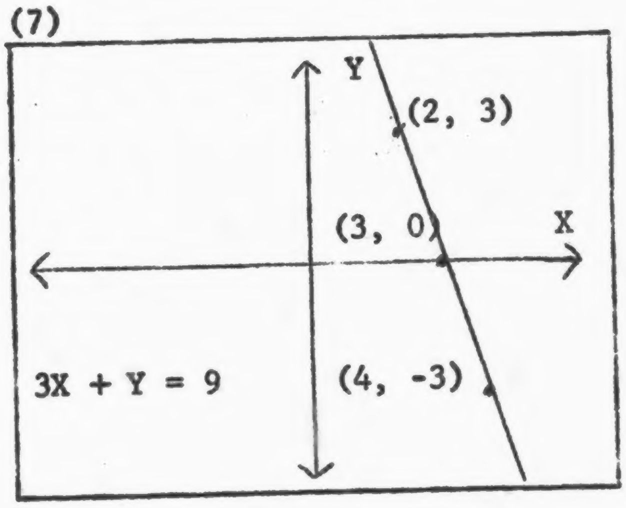
(B)

Blank Slide

(5R)



(7)

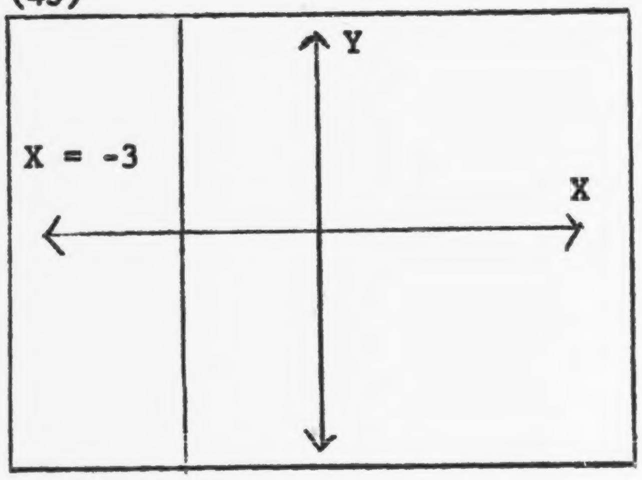

Y
(2, 3)
(3, 0)
X
3X + Y = 9
(4, -3)

RECORDED SCRIPT	WORKSHEET
(B) (B) Thirdly, equations of the form $3x + y = 9$, $x = 4$, $x - 5y = 2$ are called linear equations, because their graphs are straight lines.	
We have already found that these three number pairs, (2,3), (4,-3) and (3,0) satisfy $3x + y = 9$ (5R) and we plotted these points.	
So, to draw the graph of $3x + y = 9$, (7) we simply draw the straight line between these points. Note that the line passes through all three points.	

SCREEN

Left Half

(43)



(27)

X = -2Y

Right Half

(42A)

(-3, 4)
(-3, 3)
(-3, 2)
(-3, 1)
(-3, 0)
(-3, -1)
(-3, -2)
(-3, -3)

(B)

Blank Slide

(44)

1) Find three number pairs which satisfy the equation.

2) Plot these points.

3) Draw the straight line through them.

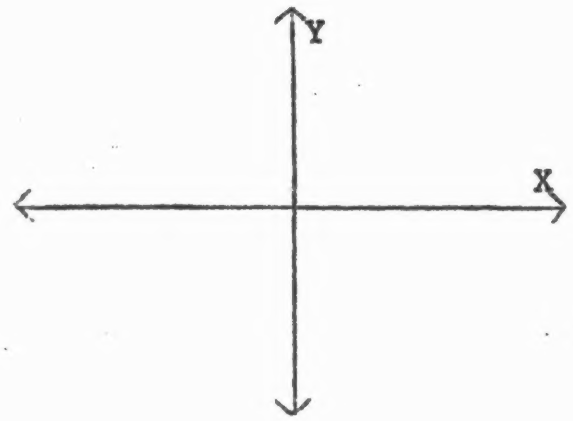
RECORDED SCRIPT

WORKSHEET

(43) Here is the graph of $x = -3$, a line parallel to the y-axis. (42A) Note that since y is not mentioned in the equation, it can have any value. All these points satisfy $x = -3$. Whenever a line is parallel to one of the axes, it has an equation of this form.

(27) (B) Let's say you are given the equation $x = -2y$ and you want to draw its graph. (44) First, find three number pairs which satisfy $x = -2y$. Then plot these points, and draw the straight line through them.*

4. Graph $x = -2y$.



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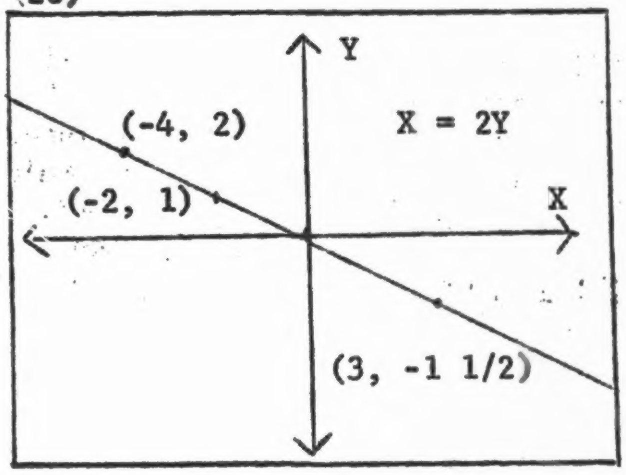
(28)


Y
X
X = 2Y
(-4, 2)
(-2, 1)
(3, -1 1/2)

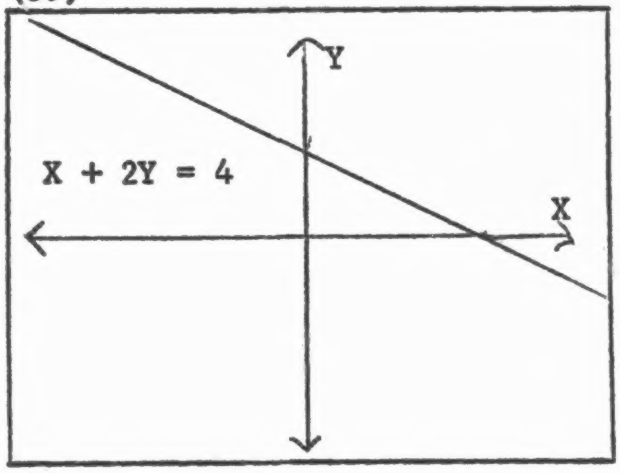
(38R)

X + 2Y = 4

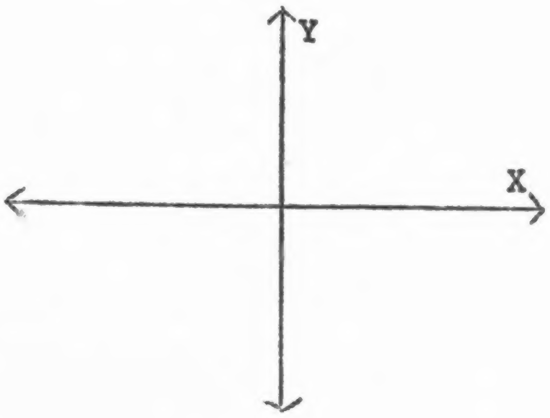
(B)

Blank Slide

(39)



RECORDED SCRIPT	WORKSHEET
(28) $x = -2y$ looks like this.	
(38R) (B) Now try graphing $x + 2y = 4$.*	5. Graph $x + 2y = 4$.



Here is the result. (39) Note that the third point serves to check our work. If the line does not go through all three points, we can go back and check our computations and the plotting of our number pairs.

GRAPHING OF INEQUALITIES:

ORIGINAL PROGRAM p' B

GRAPHING OF INEQUALITIES - p' B

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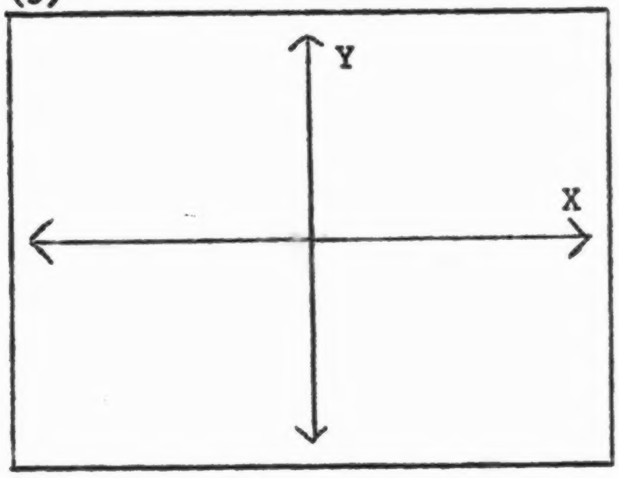
(2)

3X + Y = 9

Right Half

(3)

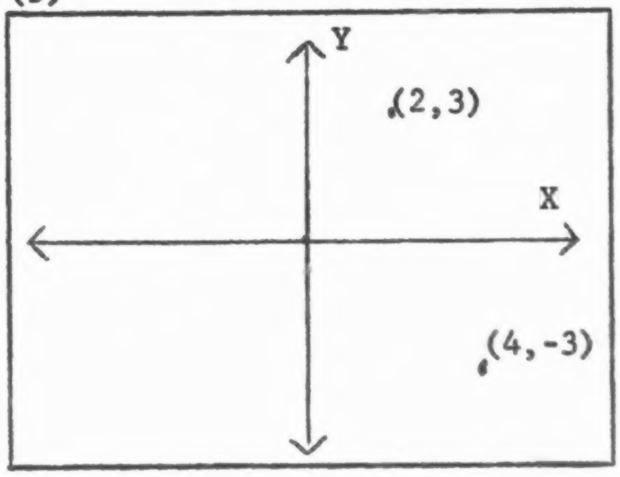


(4)

(2,3)
$3 \cdot 2 + 3 = 9$

(4,-3)
$3 \cdot 4 + -3 = 9$

(5)



RECORDED SCRIPT	WORKSHEET
You have worked with equations like $3x + y = 9$ (2) and sketched their graphs in the rational number plane. (3) You did this by finding some number pairs for which this equation was true. For instance, $3x + y = 9$ is true for (2,3) and (4,-3). (4) We plot these points. (5) You also know that any equation like this is linear so that its graph is a straight line.	

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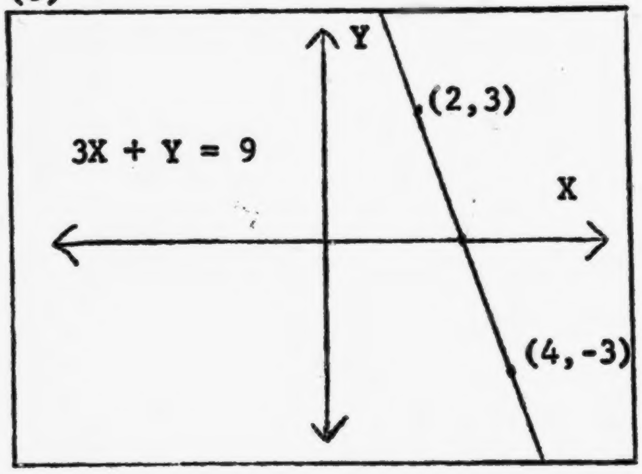
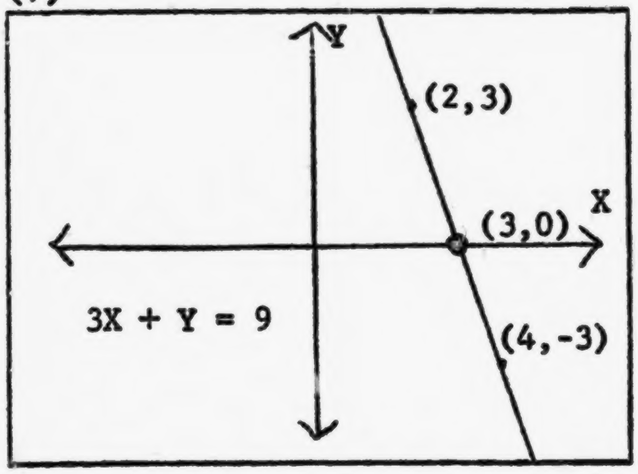
(6)



(7)



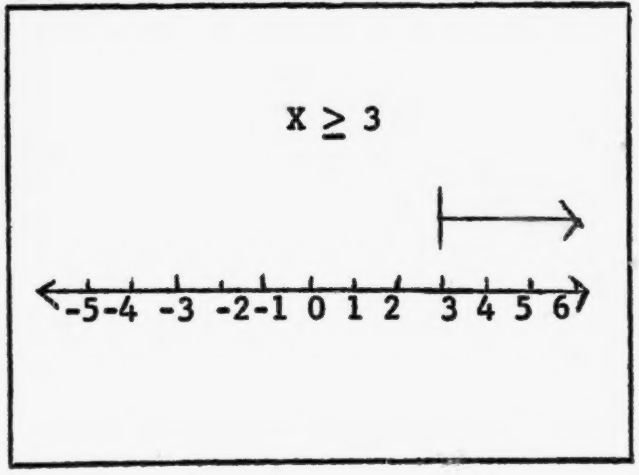
(8)

(3,0)

$3 \cdot 3 + 0 = 9$

$\therefore 9 = 9$

(9)


X ≥ 3
-5
-4
-3
-2
-1
0
1
2
3
4
5
6

(B)

Blank Slide

RECORDED SCRIPT

WORKSHEET

So once we have two points, we draw the straight line between them and we have the graph of $3x + y = 9$. (6) Is this correct? If it is, we should be able to take any point on our straight line and by substituting its coordinates in the equation, get a true statement.

Let's try it for the point (3,0). (7) Putting (8) it in our equation, we have $3 \cdot 3 + 0 = 9$ or $9 = 9$.

But an equation is only one kind of number statement. (9) (B)

SCREEN

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(B)

Blank Slide

(10)

$3X + Y \leq 9$

(10A)

$X \geq Y$

(11)

$X = Y$

RECORDED SCRIPT	WORKSHEET
For instance, you have graphed $x \geq 3$ on the number line. (B) Why couldn't we graph (10) a number statement like $3x + y \leq 9$ in the number plane?	
Such a statement is called an inequality. The graphing of inequalities has important applications for industry in linear programming, a topic we will look at later. Right now, let's find out how we might graph an inequality, for instance $x \geq y$. (10A)	
First we consider a simple equation, $x = y$. (11) List some of the ordered pairs for which $x = y$ is true.*	1. List some ordered pairs which satisfy $x = y$ (,) (,) (,)

SCREEN

Left Half **Right Half**

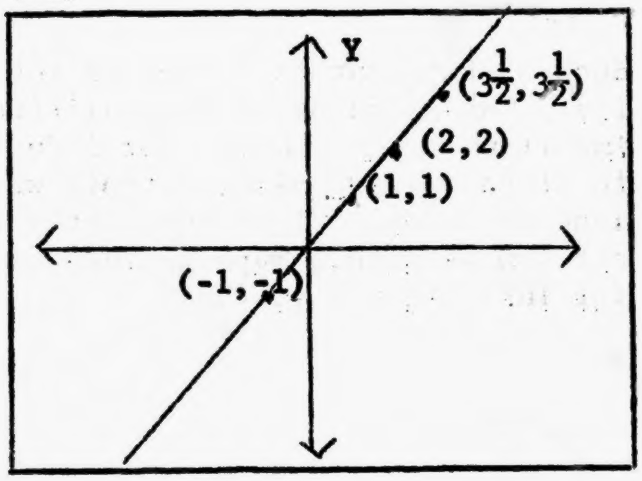
(12)

(0,0) (2,2)

(1,1) (-1/2,-1/2)

(-1,-1) $(3\frac{1}{2}, 3\frac{1}{2})$

(13)

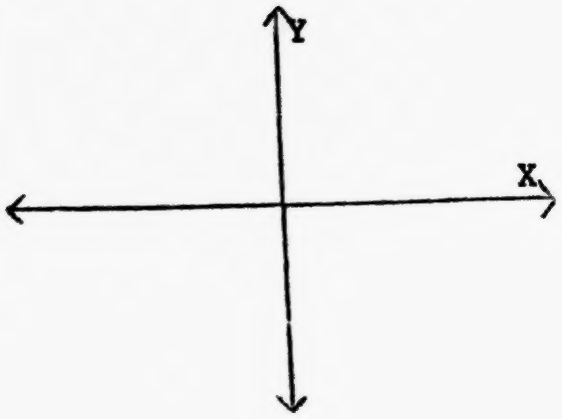

Y
$(3\frac{1}{2},3\frac{1}{2})$
(2,2)
(1,1)
(-1,-1)

(10A)

$X \geq Y$

(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
Some of the pairs you might have listed are these. (12) Now plot these points and draw the graph of $x = y$.*	2. Plot your points and draw the graph of $x = y$. 
It is a straight line (13) which goes diagonally from the lower left-hand corner to the upper right-hand corner through the origin.	
Now let's consider graphing $x \geq y$. (10A) (B) Does knowing the graph of $x = y$ help any? Can you write $x \geq y$ as two simpler statements connected by "or"?*	3. $x \geq y$ is the same ________ or ________.

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Left Half **Right Half**

(15A)

$X \geq Y$

IS THE SAME AS

$X > Y$ OR $X = Y$

(B)

Blank Slide

(17)

$X > Y$

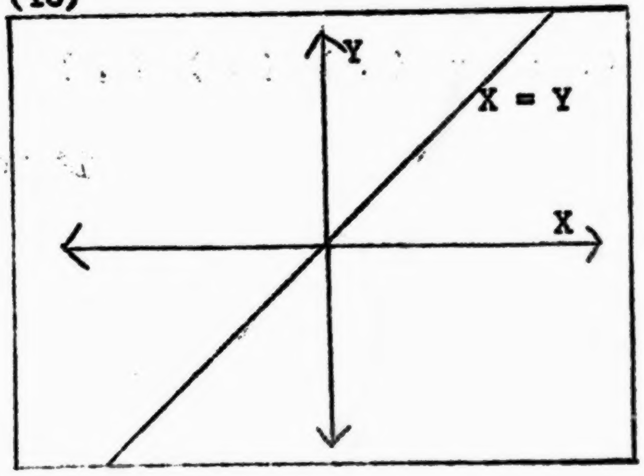
(3,2)	(1/2,1/4)
(-3,-4)	(2,-2)

RECORDED SCRIPT	WORKSHEET
(15A) Recall that $x \geq y$ is just another way of writing $x > y$ or $x = y$. We already have the graph of $x = y$. (B) List some number pairs which make $x > y$ a true statement.*	4. $x > y$ (,) (,) (,)
Here are some. (17) You may have listed others, of course. Be sure that for all the number pairs you listed, the first element is greater than the second.	

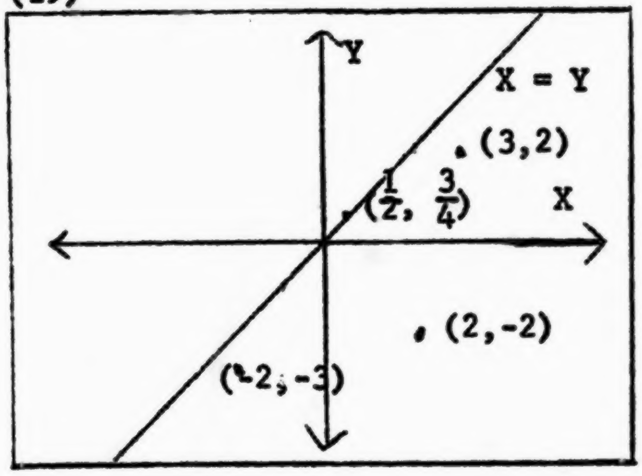
SCREEN

Left Half **Right Half**

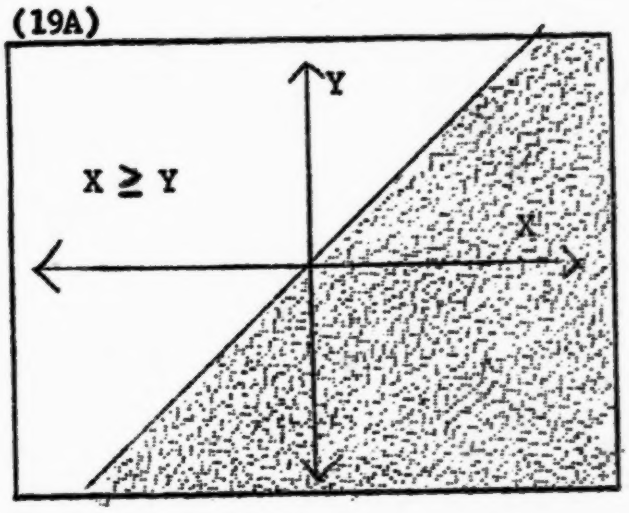
(18)



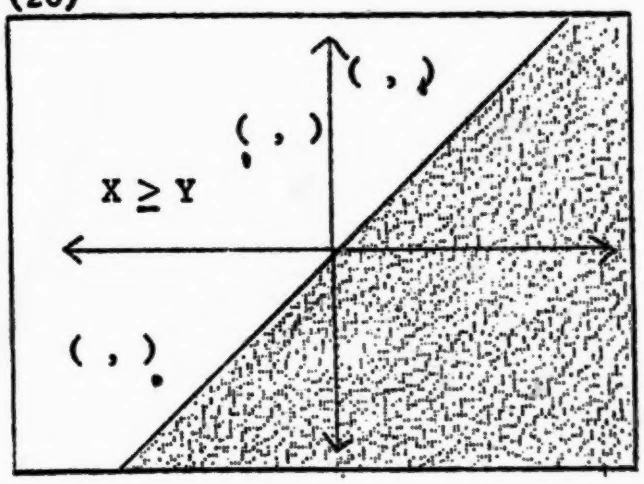
(19)


Y
X = Y
(3,2)
(1/2, 3/4)
X
(2,-2)
(-2,-3)

(19A)



(20)



(B)

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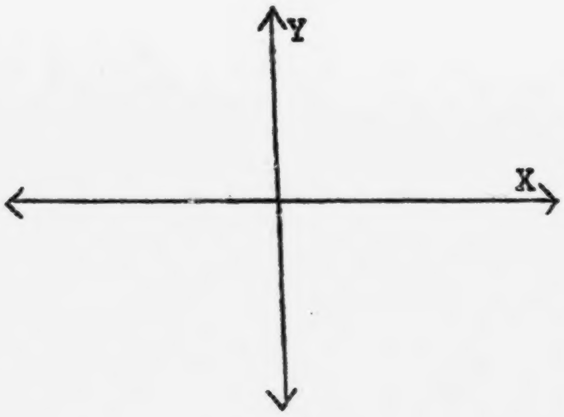
RECORDED SCRIPT	WORKSHEET
Plot the number pairs you found on the number plane. (18) Note that $x = y$ is already indicated.*	5. Plot the number pairs from the last question.
Where do all the points you plotted fall in relation to $x = y$?*	6. Where do all the points you just plotted fall? ____________________________
Here are some of the points you may have found. (19) They all lie below the line $x = y$. What do you think is the graph of $x \geq y$? Does every point below the line $x = y$ satisfy the relation $x \geq y$? (19A) Is it possible that a point above the line also belongs to the graph of $x \geq y$? Let's try a point. (20) (B)	7. Indicate the graph of $x \geq y$. 

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(21)

$(2, 3\frac{1}{2})$

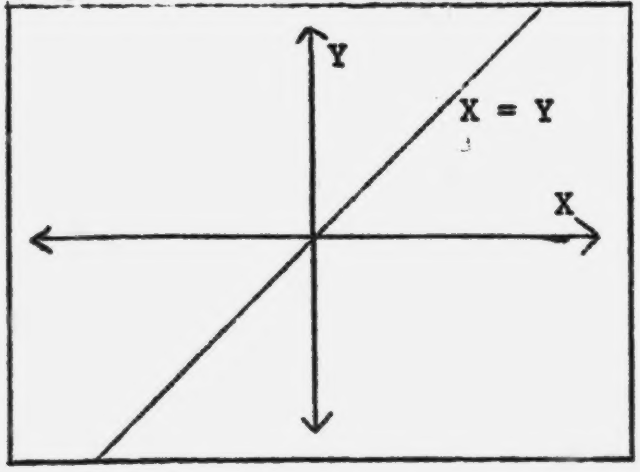
$X > Y$

Is

$2 > 3\frac{1}{2}$

a true statement ?

(22)



(B)

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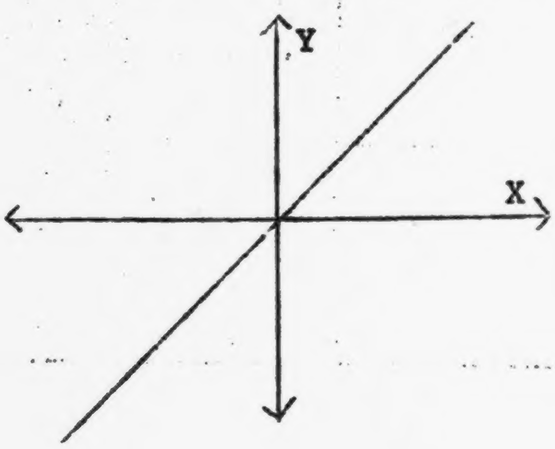
RECORDED SCRIPTS	WORKSHEET
What are the number pairs for the blue points on this graph?*	8. What are the number pairs for the blue points on this graph? (,) (,) (,)
Do they make $x \geq y$ a true statement?	9. Do the points in 8 satisfy $x \geq y$? Yes or No ________________
For instance, one of the points is the ordered pair (2, 3 1/2). (21) Here $x < y$ so it does not belong to the graph. Thus, the graph $x \geq y$ is the read shaded area.	
Here is the graph of $x = y$ again. (22) (B) Can you shade the area of $y > x$ and thus obtain the graph of $y \geq x$?*	10. Shade the graph of $y > x$. 

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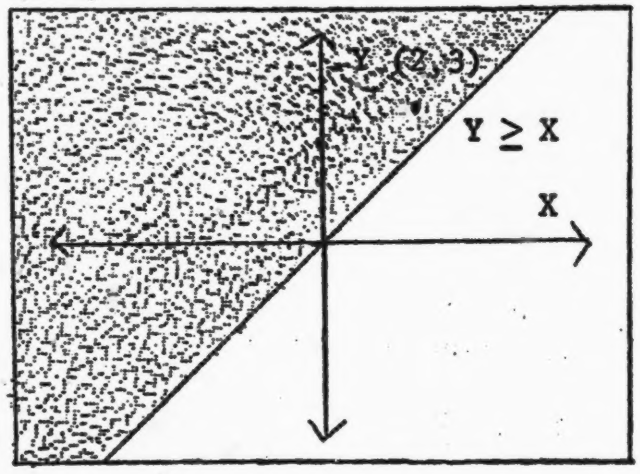
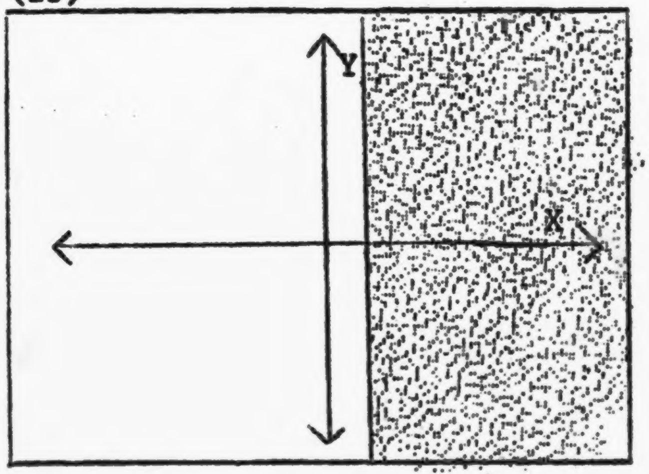
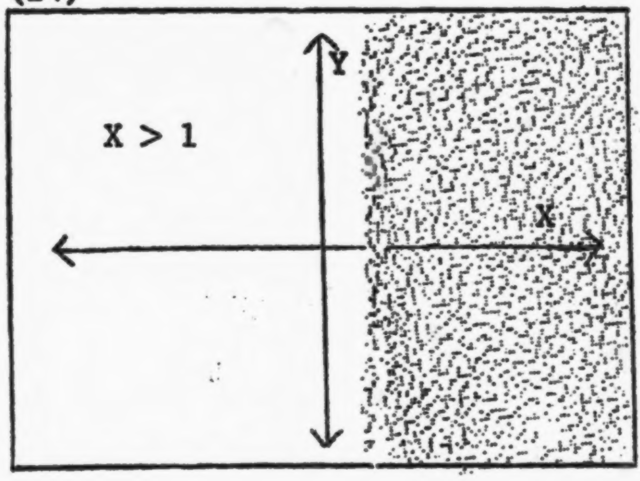
(22A)



(23)

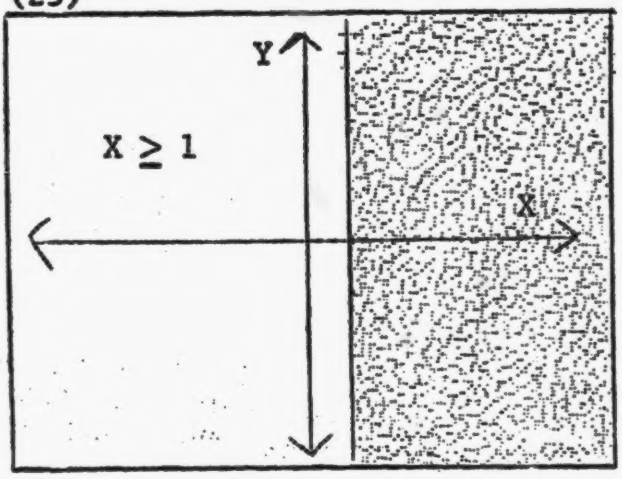


(24)



(25)

(972,1625,1498,2028)



RECORDED SCRIPTS	WORKSHEET
(22A) Here is the graph of $y \geq x$. To see this, try any number pair in the shaded area, and see that $y > x$. For instance, $3 > 2$.	
(23) The red shaded area is the graph in the plane of all ordered pairs (x,y) with $x > 1$. Is the red line a part of the graph of $x \geq 1$?* The difference between the graph of $x > 1$ (24) and the graph of $x \geq 1$ (25) is the red line $x = 1$.	11. Is the red line a part of the graph of $x \geq 1$? Yes or No __________________

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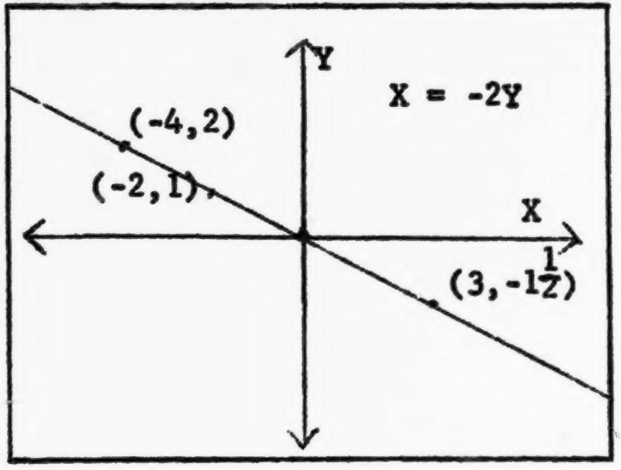
(26)	(B)
X > -2Y	Blank Slide

(27)

X = -2Y

(28)



RECORDED SCRIPT	WORKSHEET
If, then, we want to graph $x > -2y$, (26) (B) it is easier to first graph an equation. What equation would be helpful in order to get the graph of $x > -2y$?*	12. What is an equation related to $x > -2y$? ____________________
We could try $x = -2y$. (27) Find some number pairs which make $x = -2y$ a true statement, and then plot these to get a graph of $x = -2y$.*	13. a) Points which satisfy $x = -2y$. (,) (,) (,)
It looks like this. (28) Now find some ordered pairs which make $x > -2y$ a true statement. Plot them in problem 13.*	b) Points which satisfy $x > -2y$. (,) (,) (,)

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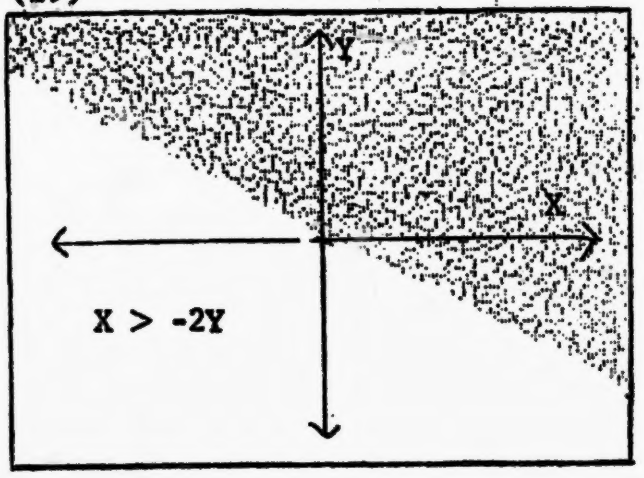
(29)



(30)

$3X + Y \geq 2$

(B)

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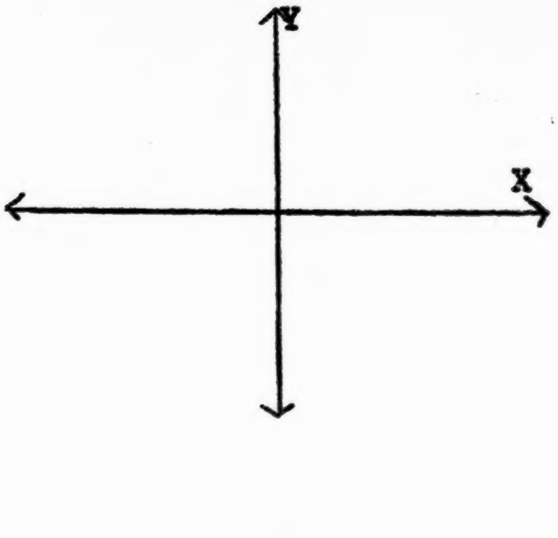
RECORDED SCRIPT	WORKSHEET
Do they all lie on one side of the line $x = -2y$?*	14. Do all the points in 13b lie on one side of the line $x = -2y$? Yes or No ______________
Is the line $x = -2y$ a part of the graph of $x > -2y$?*	15. Is the line $x = -2y$ a part of the graph of $x > -2y$? Yes or No ______________
(29) When you use a related equation like $x = -2y$, why don't you include its graph as a part of the graph of $x > -2y$?*	16. Why? ______________ ______________
Now see if you can graph $3x + y \geq 2$. (30) (B)	17. $3x + y \geq 2$. 

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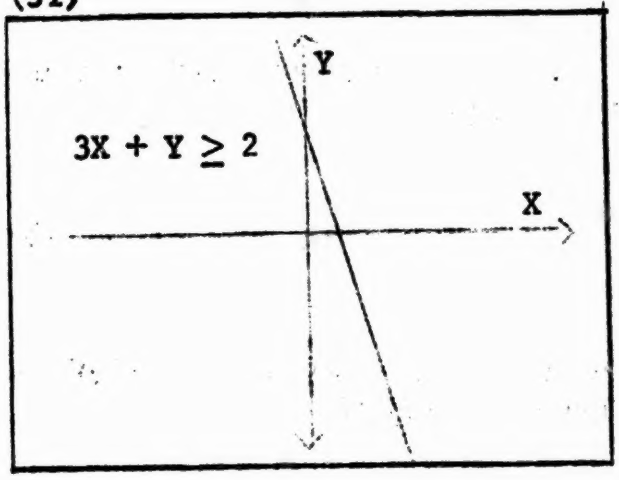
(31)



RECORDED SCRIPT	WORKSHEET
(31) Here is the correct graph. Note that since the equation is $3x + y > 2$ or $3x + y = 2$, the line $3x + y = 2$ is included as part of the graph.	
(B) (B) We will explore this kind of problem further and see how the graphing of inequalities can be used to solve some interesting problems.	

GRAPHING OF INEQUALITIES:

FIRST REVISION p'' B

GRAPHING OF INEQUALITIES - p'' B

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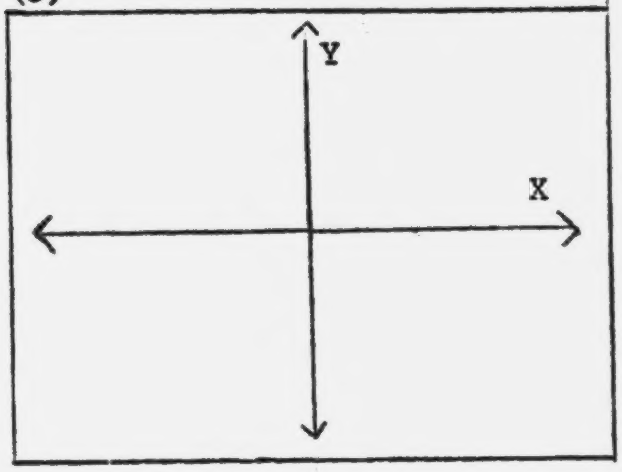
(2)

$3X + Y = 9$

Right Half

(3)



(8)

(3,0)

$3 \cdot 3 + 0 = 9$

$\therefore 9 = 9$

(B)

Blank Slide

(12)

(0,0)	(2,2)
(1,1)	(-1/2,-1/2)
(-1,-1)	$(3\frac{1}{2}, 3\frac{1}{2})$

RECORDED SCRIPT	WORKSHEET
In order to have a picture of an equation (2) such as $3x + y = 9$, we often graph such an equation in the rational number plane. (3) What does it mean to graph an equation, and how do we go about doing it?	
First, what do we mean when we say that the number pair (3,0) satisfies the equation $3x + y = 9$? We mean that if we substitute 3 for x and 0 for y in $3x + y = 9$, we get a true statement, (8) since $3 \times 3 + 0 = 9$.	
(B) Which of these ordered pairs (12) satisfy $x = y$?*	1. Which of the ordered pairs shown satisfy $x = y$? ____________________

SCREEN

Left Half	Right Half
(36) DOES (-2,1) SATISFY $3X + Y = 9$?	(B) Blank Slide
	(37) $3X + Y \overset{?}{=} 9$ $3 \cdot -2 + 1 \overset{?}{=} 9$ $-5 \neq 9$

RECORDED SCRIPT	WORKSHEET
They all do, for the first element of each order pair is equal to the second.	
Does (36) (B) the point (-2,1) satisfy $3x + y = 9$? Try it.*	2. Does (-2,1) satisfy $3x + y = 9$? ________________________ ________________________ ________________________ Yes or No ____________
(-2,1) does not satisfy $3x + y = 9$ because (37) when we substitute it in the equation, we get the result $-5 = 9$, certainly not a true statement.	

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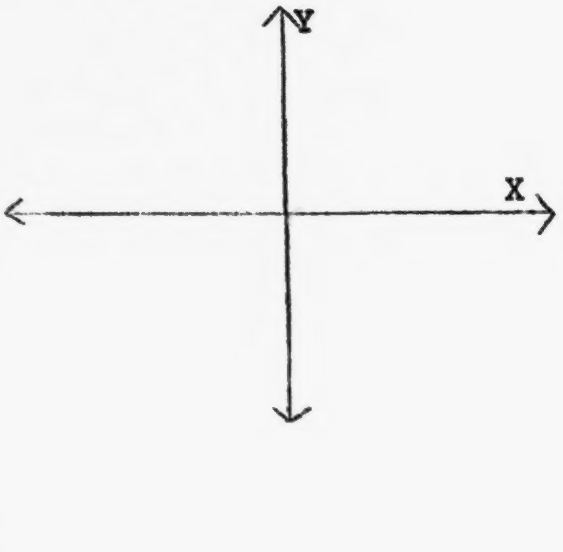
Left Half	Right Half
(4R) (2,3) (4,-3)	**(2)** $3X + Y = 9$
	(4) (2,3) $3 \cdot 2 + 3 = 9$ (4,-3) $3 \cdot 4 + -3 = 9$
(8R) (3,0)	**(B)** Blank Slide

RECORDED SCRIPT	WORKSHEET
Check (4R) (2) (2,3) and (4,-3) to see if they satisfy 3x + y = 9.*	3. Do (2,3), (4,-3) satisfy 3x + y = 9? __________ __________ __________ __________ __________ __________ (2,3) Yes or No ____ (4, -3) Yes or No ____
Here are (4) the results of substituting the points into the equation. Note that in both cases we get a true statement.	
Secondly, (8R) (B) let's take a look at a number pair, for instance (3,0). The first coordinate, in yellow, also called the x-coordinate, is three. The second or y-coordinate, in green, is zero. So, to plot this number pair, start at the origin where the x and y axes cross, and go three units to the right. Then go zero units up. Do this.*	4. Plot (3,0). 

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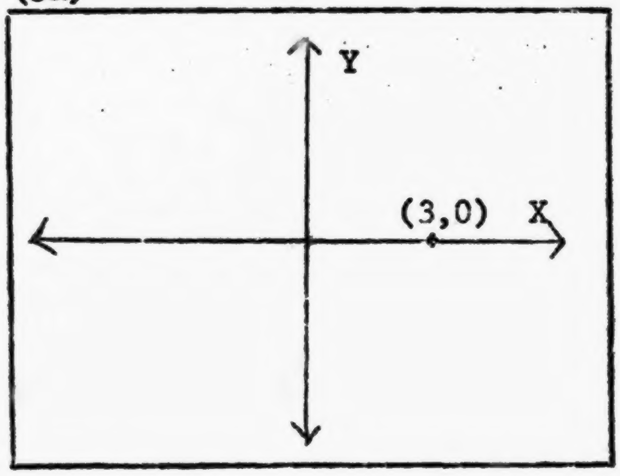
(3R)



(B)

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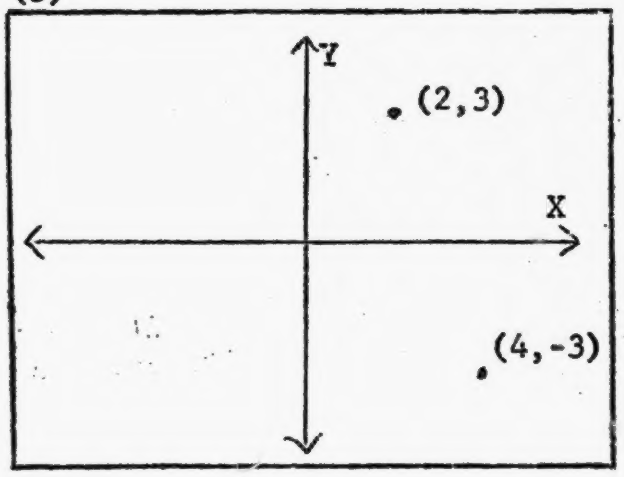
(B)

Blank Slide

(5)



RECORDED SCRIPT	WORKSHEET
(3R) Here is the result.	
Try plotting these number pairs, (B) (B) (2,3) and (4,-3).*	5. Plot (2,3) and (4,-3).
The (5) plotted pairs look like this. Notice that for the pair (4,-3) after going 4 units to the right, you go down three units because the y-coordinate is negative three.	

GRAPHING OF INEQUALITIES - p'' B

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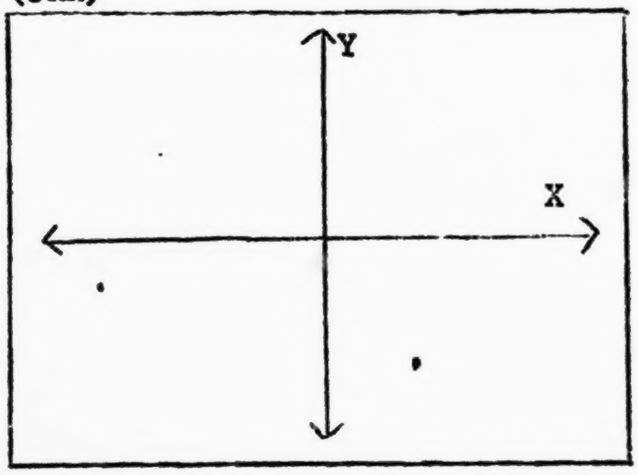
Left Half **Right Half**

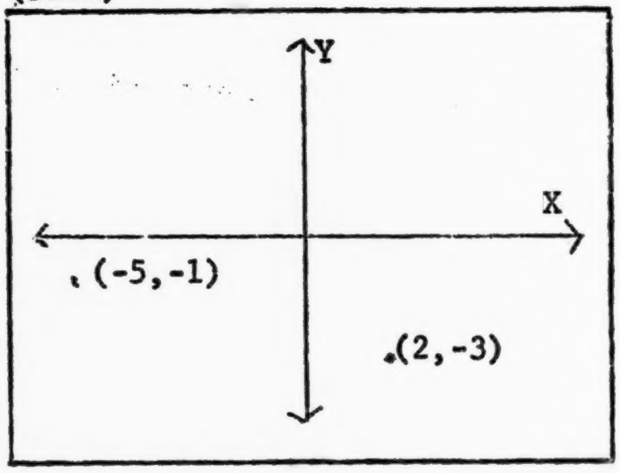
(3RR)



(B)

Blank Slide

(3RRR)



(2)

$3X + Y = 9$

(B)

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RECORDED SCRIPT	WORKSHEET
(3RR) (B) Let's turn the problem around. Here are two points plotted on the rational number plane. Can you write down the coordinates of each point by seeing first how many units in the x-direction you must go, and then how many units in the y-direction?*	6. The two points are: (,) (,)
The point (3RRR) on the left is (-5,-1) the other point is (2,-3).	
Thirdly, (B) we note the fact that equations of the form 3x + y = 9 (2) where we have only x's, y's and numbers, are called linear, because they always graph as straight lines.	

GRAPHING OF INEQUALITIES - p'' B

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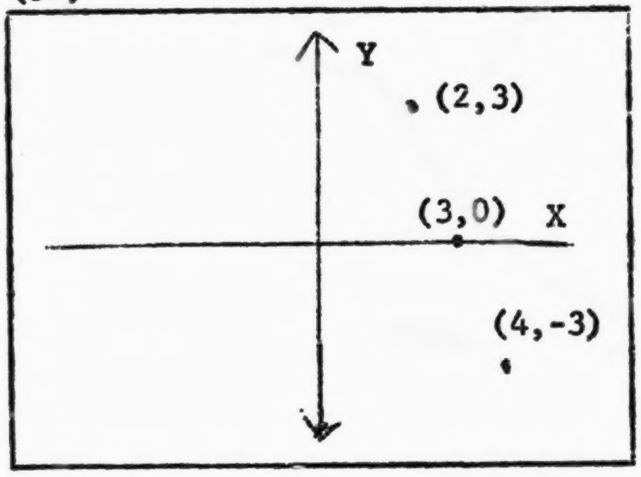
(42)

$X = -3$

(38)

$X + 2Y = 6$

(5R)



(B)

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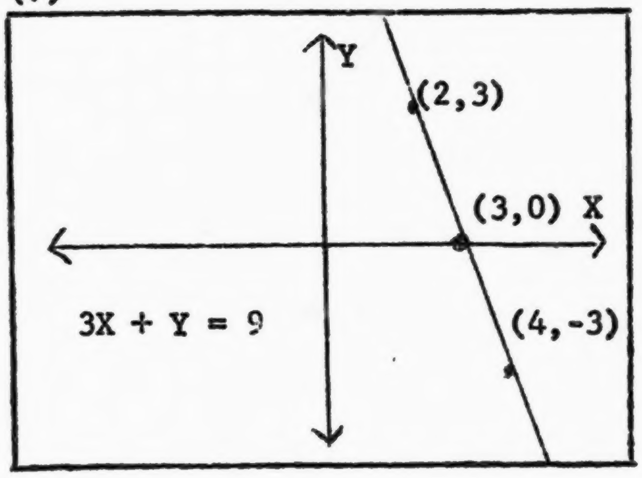
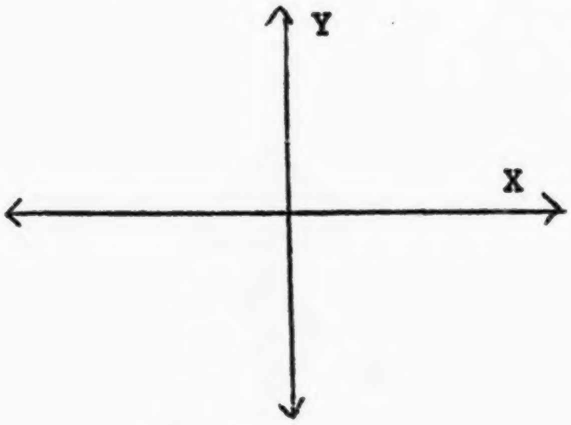
(7)


Y
(2,3)
(3,0)
X
3X + Y = 9
(4,-3)

RECORDED SCRIPTS	WORKSHEET
$x = -3$ (42) and $x + 2y = 6$ (38) are other examples of linear equations.	
(B) For the equation $3x + y = 9$ we have found and plotted three number pairs which satisfy the equation. (5R) Here are these points. To draw the graph of $3x + y = 9$, we simply draw the straight line between these points. Do this. Does the line pass through all three points?*	7. Draw the graph of $3x + y = 9$. 
(7) The graph of $3x + y = 9$ is this straight line which passes through all three points we plotted.	

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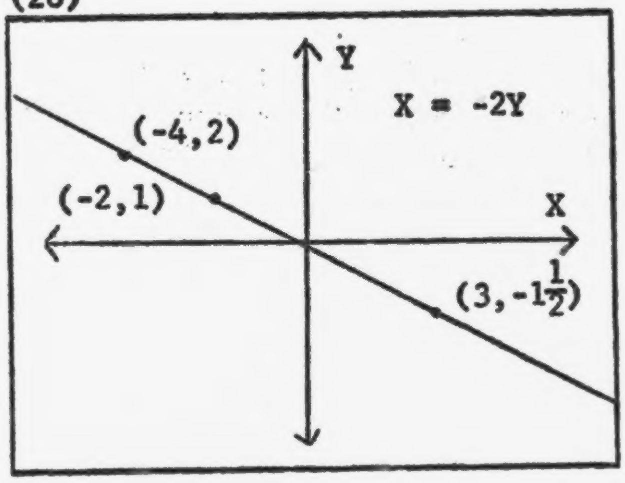
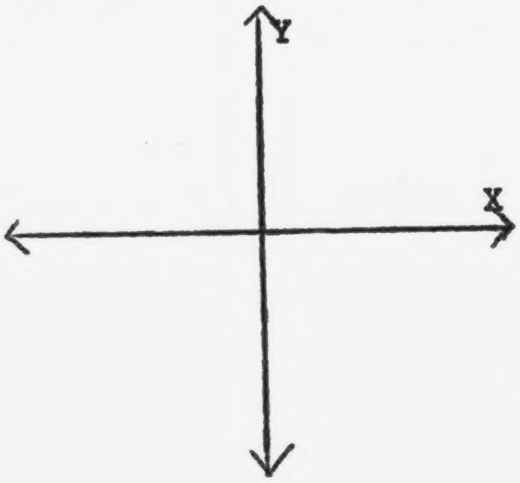
(27)

X = -2Y

(28)



RECORDED SCRIPT	WORKSHEET
(27) Let's say you are given the equation $x = -2y$ and you want to draw its graph. Let's go through the three steps once more. First, how will you find points which satisfy $x = -2y$? Try picking a value for y, say $y = 2$. What does x equal if $y = 2$?*	8. a.) If $y = 2$, what is x? ______ The number pair is (, 2).
$x = -4$, so the number pair is (-4,2).	
What is x if $y = 0$?*	b.) If $y = 0$, what is x? ______ The number pair is (, 0).
$x = 0$ and the number pair (0,0) satisfies our equation.	
What is x if $y = -1\ 1/2$?*	c.) If y is $-1\ 1/2$, what is x? ______________ The number pair is (,, -1 1/2).
x is 3, and the number pair is (3, -1 1/2).	
So we have found three points which satisfy our equation. The second step is to plot these points. Do this.*	9. Plot the three number pairs in question 8. 
Finally draw a straight line through these points.	
(28) Did your line pass through all three points like this? If not, recheck the way you plotted the points.	

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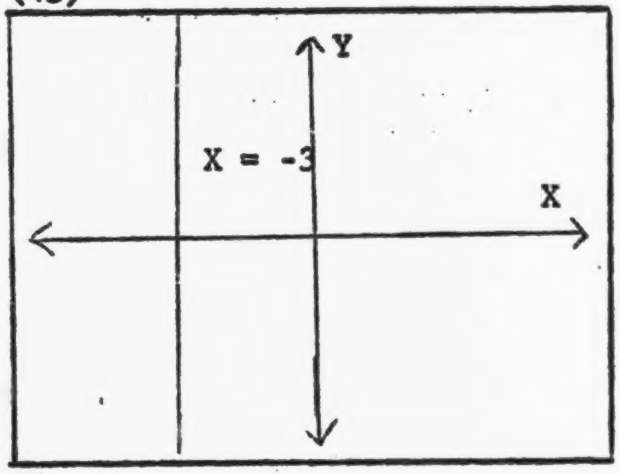
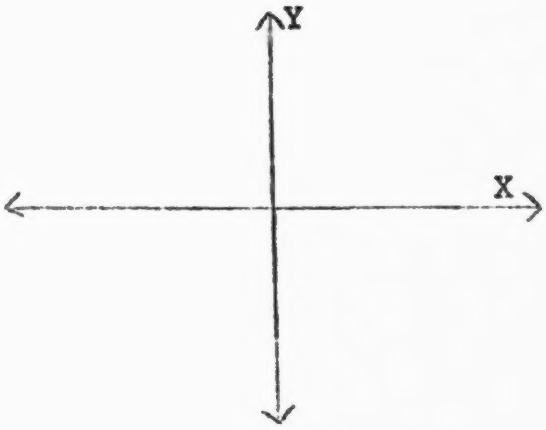
Left Half	Right Half
(42) X = -3	(B) Blank Slide
	(43) 
(38R) X + 2Y = 4	(B) Blank Slide

RECORDED SCRIPT	WORKSHEET
(42) (B) Let's look at the equation x = -3. List some number pairs which make x = -3 a true statement.*	10. List some number pairs which satisfy x = -3. (,) (,) (,)
You could have listed any number pairs with -3 as the first element. For instance (-3,0) or (-3,2) or (-3,-4). Since y is not mentioned in the equation, it can have any value. Plot your points.* Do they all lie on a straight line like this?* (43)	11. Plot your points from 10. 
Now try (38R) (B) graphing x + 2y = 4. If you are stuck as to how to begin, first solve the equation for x.*	12. Solve x + 2y = 4 for x. ____________________

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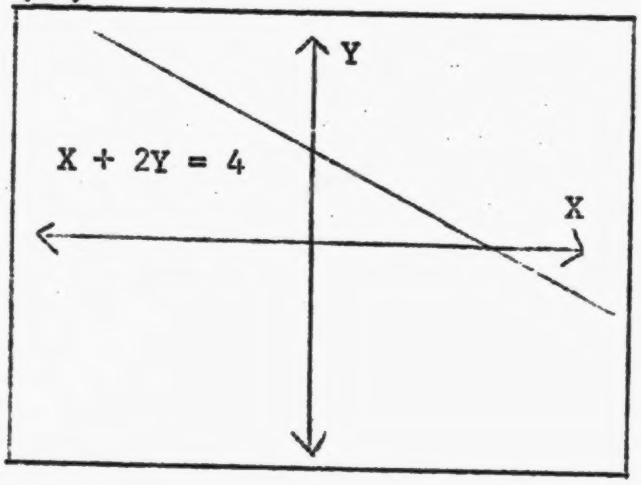
(39)



RECORDED SCRIPTS	WORKSHEET
$x = 4 - 2y$ is just another way of writing $x + 2y = 4$. Now you can pick 3 values for y. and find the corresponding values for x. Do this; plot your points, and draw the straight line joining them.*	13. $x = 4 - 2y$ (x,y) (,) (,) (,)
(39) Here is the result. Note again that the third point serves to check our work. If the line does not go through all three points, we can go back and check our computations and the plotting of our number pairs.	

GRAPHING OF INEQUALITIES:

SECOND REVISION p''' B

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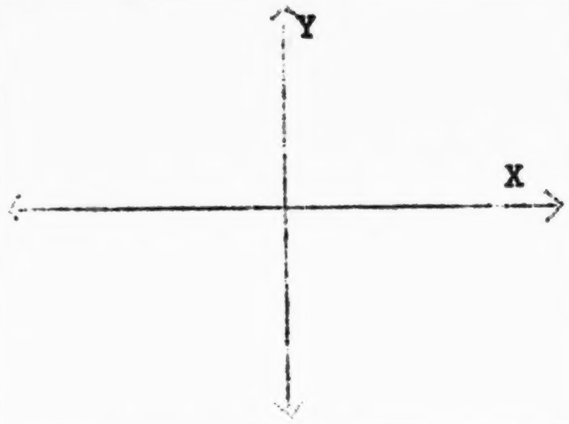
(2)

$3X + Y = 9$

(3)



(8)

(3,0)

$3 \cdot 3 + 0 = 9$

$\therefore 9 = 9$

(B)

Blank Slide

(12)

(0,0) (2,2)

(1,1) (-1/2,-1/2)

(-1,-1) $(3\frac{1}{2}, 3\frac{1}{2})$

RECORDED SCRIPT	WORKSHEET
In order to have a picture of an equation (2) such as $3x + y = 9$, we often graph such an equation in the rational number plane. (3) What does it mean to graph an equation, and how do we go about doing it?	
First, what do we mean when we say that the number pair (3,0) satisfies the equation (8) $3x + y = 9$. We mean that if we substitute 3 for x and 0 for y in $3x + y = 9$, we get a true statement, since $3 \times 3 + 0 = 9$.	
(B) Which of these ordered pairs (12) satisfy $x = y$?*	1. Which of the ordered pairs shown satisfy $x = y$? ____________________
They all do, for the first element of each ordered pair is equal to the second.	

SCREEN

Left Half | **Right Half**

(36)

DOES
(-2,1)
SATISFY

$3X + Y = 9$?

(B)

Blank Slide

(37)

$$3X + Y \overset{?}{=} 9$$

$$3 \cdot -2 + 1 \overset{?}{=} 9$$

$$-5 \neq 9$$

(4R)

(2,3)

(4,-3) ?

(2)

$3X + Y = 9$

RECORDED SCRIPT	WORKSHEET
Does (36) (B) the point (-2,1) satisfy $3x + y = 9$? Try it.*	2. Does (-2,1) satisfy $3x + y = 9$? ____________________ ____________________ ____________________ Yes or No ____________
(-2,1) does not satisfy $3x + y = 9$ because (37) when we substitute it in the equation, we get the result $-5 = 9$, certainly not a true statement.	
Check (4R) (2) (2,3) and (4,-3) to see if they satisfy $3x + y = 9$.*	3. Do (2,3), (4,-3) satisfy $3x + y = 9$? __________ __________ __________ __________ __________ __________ (2,3) Yes or No ____ (4,-3) Yes or No ____

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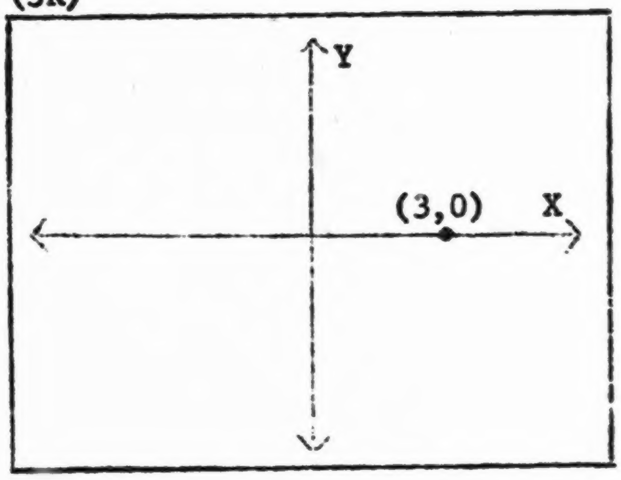
	(4) (2,3) $3 \cdot 2 + 3 = 9$ (4,-3) $3 \cdot 4 + -3 = 9$
(8R) (3,0)	**(B)** Blank Slide
	(3R)



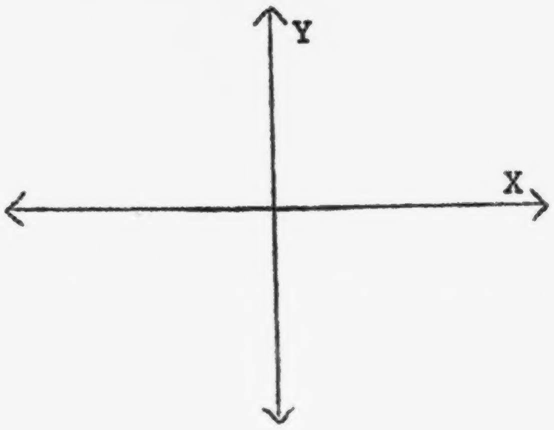
RECORDED SCRIPT

WORKSHEET

Here are (4) the results of substituting the points into the equation. Note that in both cases we get a true statement.

Secondly, (8R) (B) let's take a look at a number pair, for instance (3,0). The first coordinate, in yellow, also called the x-coordinate, is three. The second or y-coordinate, in green, is zero. So, to plot this number pair, start at the origin where the x - and y-axes cross, and go three units to the right. Then go zero units up. Do this.*

4. Plot (3,0).



(3R) Here is the result.

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(B)

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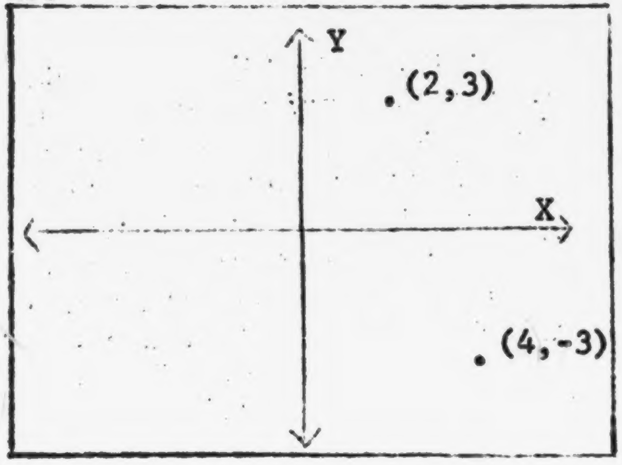
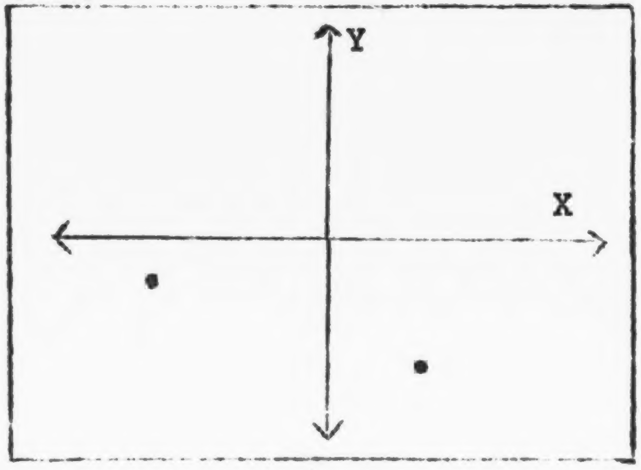
(B)

Blank Slide

(5)



(3RR)



(B)

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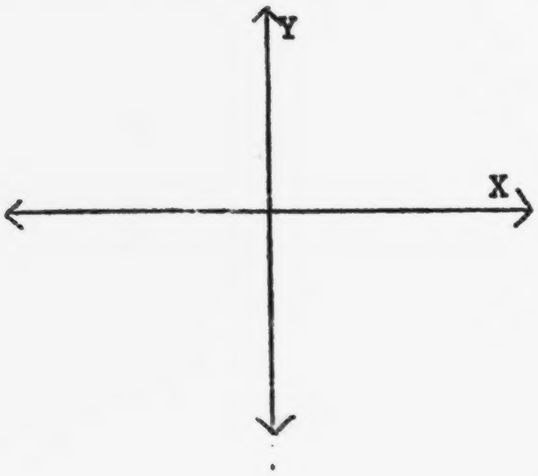
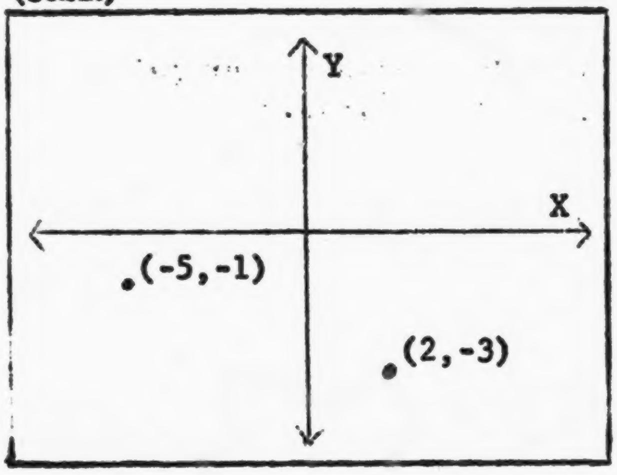
RECORDED SCRIPT	WORKSHEET
Try plotting these number pairs, (B) (B) (2,3) and (4,-3).*	5. Plot (2,3) and (4,-3). 
The (5) plotted pairs look like this. Notice that for the pair (4,-3) after going 4 units to the right, you go down three units because the y-coordinate is negative three.	
(3RR) (B) Let's turn the problem around. Here are two points plotted on the rational number plane. Can you write down the coordinates of each point by seeing first how many units in the x-direction you must go, and then how many units in the y-direction?*	6. The two points are: (,) (,)

SCREEN

Left Half	Right Half
	(3RRR)



(2)

$3X + Y = 9$

(B)

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(42)

$X = -3$

(38)

$X + 2Y = 6$

RECORDED SCRIPT

WORKSHEET

The point (3RRR) on the left is (-5,-1) the other point is (2,-3).

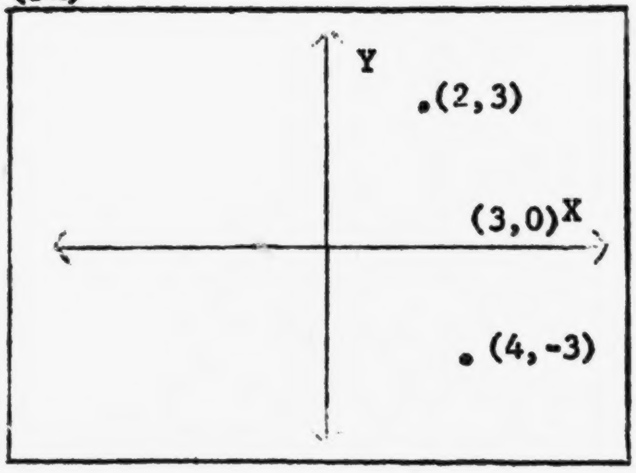
Thirdly, (B) we note the fact that equations of the form 3x + y = 9 (2) where we have only x's, y's and numbers, are called linear, because they always graph as straight lines.

x = 3 (42) and x + 2y = 6 (38) are other examples of linear equations.

SCREEN

Left Half	Right Half
(5R)	(B)

(5R)



(B)

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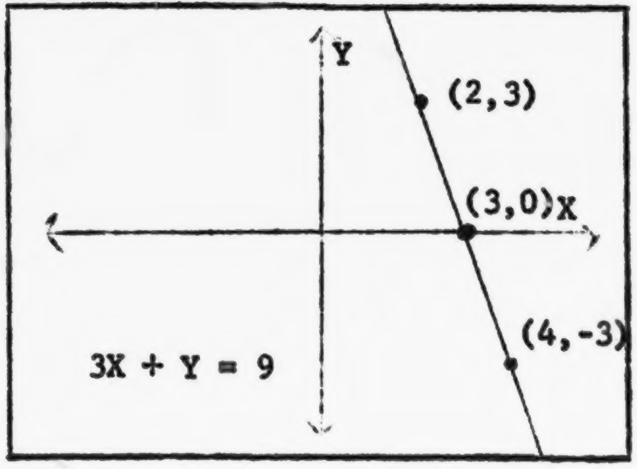
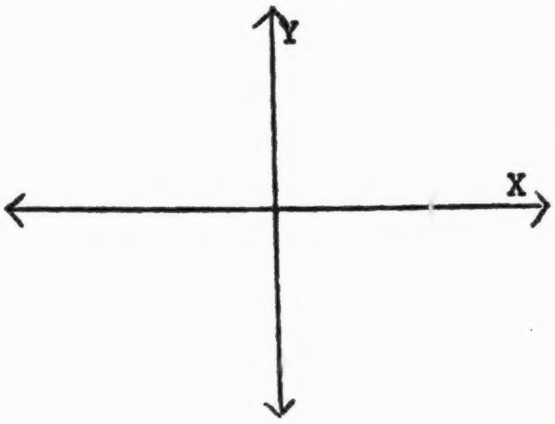
(7)


Y
(2,3)
(3,0)
X
(4,-3)
3X + Y = 9

(27)

$X = -2Y$

RECORDED SCRIPT	WORKSHEET
(B) For the equation $3x + y = 9$, we have found and plotted three number pairs which satisfy the equation. (5R) Here are these points. To draw the graph of $3x + y = 9$, we simply draw the straight line between these points. Do this. Does the line pass through all three points?*	7. Draw the graph of $3x + y = 9$. 
(7) The graph of $3x + y = 9$ is this straight line which passes through all three points we plotted. The graph is not just the line segment joining these three points, but the line extending infinitely far in each direction.	
(27) Let's say you are given the equation $x = -2y$ and you want to draw its graph. Let's go through the three steps once more. First, how will you find points which satisfy $x = -2y$? Try picking a value for y, say $y = 2$. What does x equal if $y = 2$?* $x = -4$, so the number pair is $(-4,2)$.	8. a.) If $y = 2$, what is x? ______ The number pair is (, 2).

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Left Half	Right Half

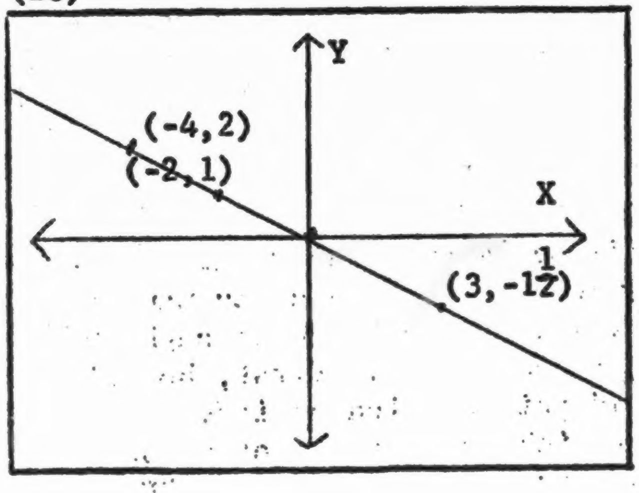
(27A)

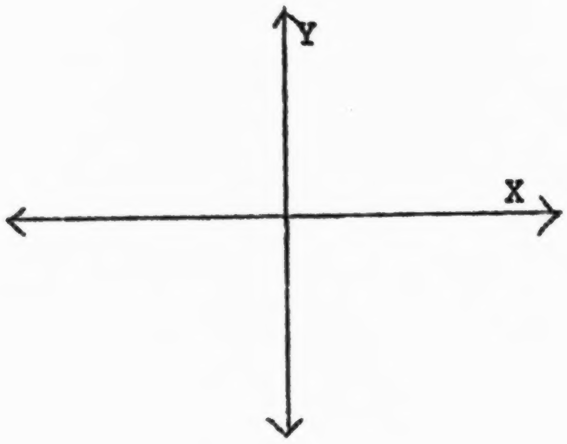
$(-4,2)$

$(0,0)$

$(3,-1\frac{1}{2})$

(28)



RECORDED SCRIPTS	WORKSHEET
What is x if y = 0?*	b.) If y = 0, what is x? ______
x = 0 and the number pair (0,0) satisfies our equation.	The number pair is (, 0).
What is x if y = -1 1/2?	c.) If y is -1 1/2, what is x? ______
x is 3, and the number pair is (3, -1 1/2).	The number pair is (,$-1\frac{1}{2}$)
(27A) So we have found three points which satisfy our equation. The second step is to plot these points. Do this.*	9. Plot the number pairs in question 8. 
Finally, draw a straight line through these points.*	
(28) Did your line pass through all three points like this? If not, recheck the way you plotted the points.	

GRAPHING OF INEQUALITIES - p''' B

SCREEN

Left Half

(42)

X = -3

Right Half

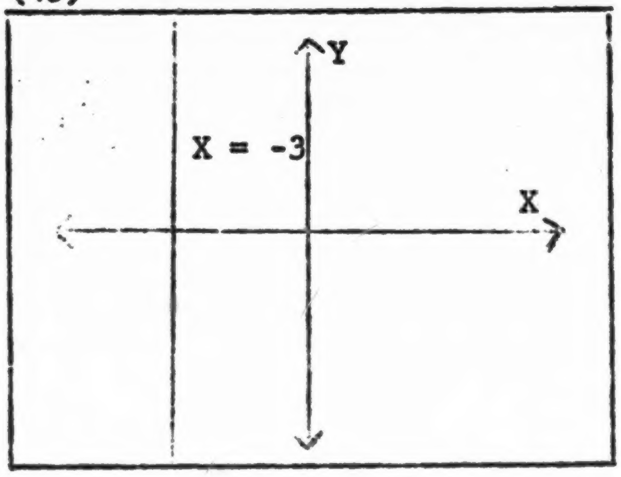
(B)

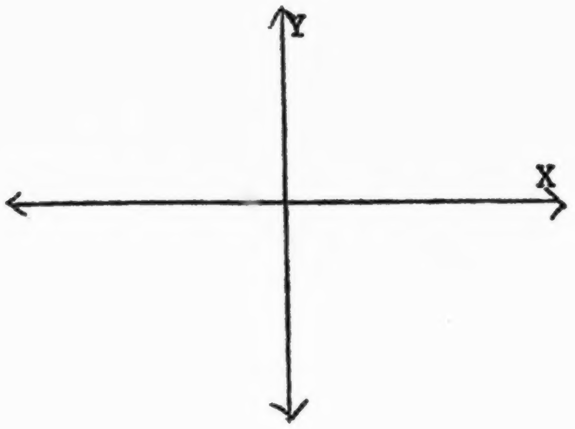
Blank Slide

(42A)

(-3,4)
(-3,3)
(-3,2)
(-3,1)
(-3,0)
(-3,-1)
(-3,-2)
(-3,-3)

(43)



RECORDED SCRIPT	WORKSHEET
(42) (B) Let's look at the equation $x = -3$. List some number pairs which make $x = -3$ a true statement.*	10. List some number pairs which satisfy $x = -3$.
(42A) You could have listed any number pairs with -3 as the first element. For instance (-3,0) or (-3,2) or (-3,-4). Since y is not mentioned in the equation, it can have any value. Plot your points.* Do they all lie on a straight line like this? (43)	11. Plot your points from question 10. 

SCREEN

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Right Half

(38R)

$X + 2Y = 4$

(B)

Blank Slide

(39A)

$X = 4 - 2Y$

X	Y

(39A)

$X = 4 - 2Y$

X	Y

RECORDED SCRIPT

WORKSHEET

Now try (38R) (B) graphing x + 2y = 4. If you are stuck as to how to begin, first solve the equation for x.*

12. Solve x + 2y = 4 for x.

____

(39A) x = 4 - 2y is just another way of writing x + 2y = 4.

Now you can pick 3 values for y and find the corresponding values for x. (39A) You might organize your information in the form of a chart like this one on the screen. Do this, plot your points, and draw the straight line joining them.*

13. x = 4 - 2y.

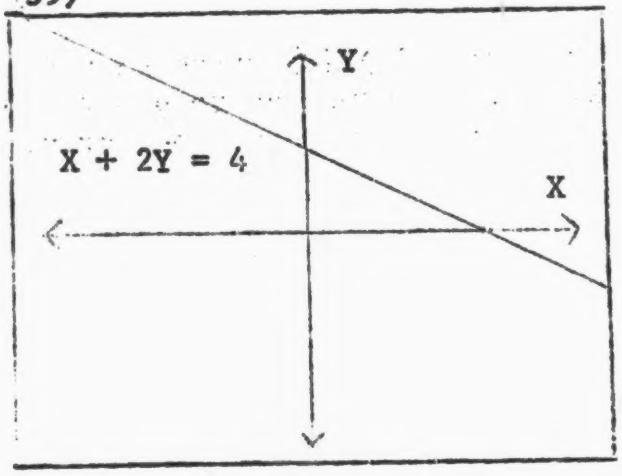
x	y

SCREEN

Left Half

Right Half

(39)



(40)

X - Y = 3

(B)

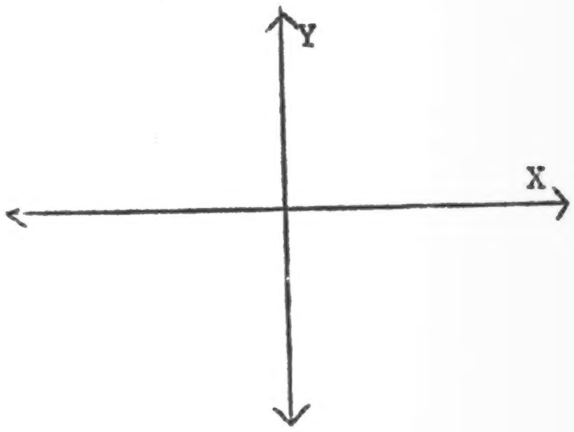
Blank Slide

(44)

1) Find three number pairs which satisfy the equation.

2) Plot these points.

3) Draw the straight line through them.

RECORDED SCRIPTS	WORKSHEET
(39) Here is the result. Note again that the third point serves to check our work. If the line does not go through all three points, we can go back and check our computations and the plotting of our number pairs.	
(40) (B) Graph x - y = 3. Follow the (44) three steps as outlined on the screen.*	14. 

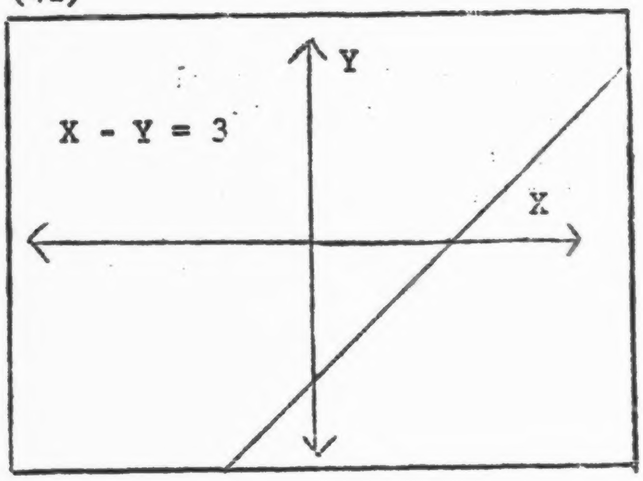
GRAPHING OF INEQUALITIES - p''' B

SCREEN

Left Half

Right Half

(41)



RECORDED SCRIPTS	WORKSHEET
(41) The graph is this line.	

GRAPHING OF INEQUALITIES:
MEASURE Y_1

ANSWERS:

1. Plot and label the following points.

   (3,0)
   (-2,-2)
   $(-2\frac{1}{2},1)$

2. Graph $2x + 3y = 1$

3. Break the number statement $5x + 3y \geq 4$ up into an equation and an inequality:

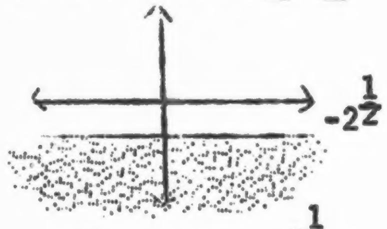
   $5x + 3y \geq 4$ is the same as ________

   ____ ______________________

   3. $5x + 3y > 4$ or
   $5x + 3y = 4$

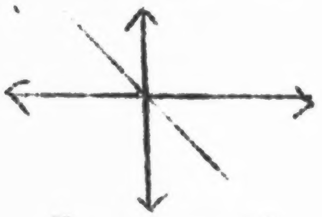
4. Complete the equation of each of these graphs by filling in the blank with the correct sign (<, >, =, ≤, ≥)

(a)



y ______ $-2\frac{1}{2}$

(b)



x ______ -y

4. a) $y < -2\frac{1}{2}$

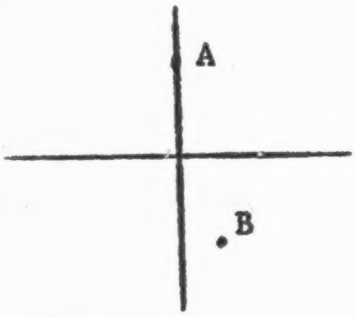
   b) $x = -y$

5. Graph:
   (a) $x > -2$

   (b) $x + y \leq 6$

Point distribution: $1\text{-}1\frac{1}{2}$, 2-1, $3\text{-}1\frac{1}{2}$, 4-2, 5-4.
Total 10.

GRAPHING OF INEQUALITIES:
MEASURE Y_2

	ANSWERS:
1. Plot and L A B E L each number pair. a.) (-2,-2) b.) (-3,0) c.) (5,-4) d.) (0,1)	
2. Does (2,-7) satisfy $4y - x = 1$?	2. No
3. Does (-3,1) satisfy $3x + 2y = -7$?	3. Yes
4. Write down the number pair for each point. 	4. A (0,3) B (2,-3)
5. Graph $x = 2y$ Graph $3x + 2y = 6$	
6. Write the equation of the red line.	6. $x = 2$

Point distribution: 1-2, 2-1, 3-1, 4-1, 5-4, 6-1.
Total 10.

GRAPHING OF INEQUALITIES
MEASURE Y_3

ANSWER:

1. Plot and LABEL each number pair.
   - a.) $(0, -3\frac{1}{2})$
   - b.) $(-4, 2)$
   - c.) $(5, 0)$
   - d.) $(3, -3)$

2. The point (a,b) satisfies $2x \quad y = 5$.
   If $a = 1$, what is b?

$b = 3$

3. Does $(1, -3)$ satisfy $3y \quad 2x = -5$?

No

4. Write down the number pair for each point.

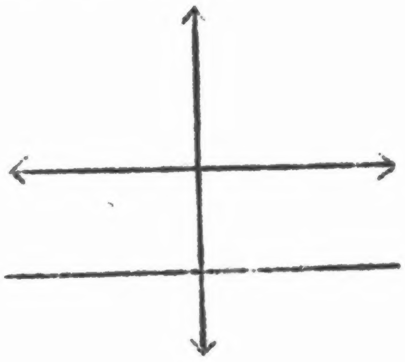
a. $(-5, 0)$
b. $(3, 4)$

5. Graph $x = 4y$

Graph $3y \quad 2x = 12$

6. Write the equation of the red line.

$y = 3\frac{1}{2}$



Point distribution: 1-2, 2-1, 3-1, 4-1, 5-4, 6-1.
Total 10.

INEQUALITIES AND THE NUMBER PLANE

Introduction to the *Inequalities and the Number Plane* Program

Inequalities and the Number Plane was an outgrowth of the *Graphing of Inequalities* program, which was too long and assumed too much as far as the ability of a student to graph straight lines was concerned. This program teaches just the concept of an inequality, based on the knowledge taught in the previous program of the graphing of a straight line. It does not consider what was regarded as an additional concept, the distinction between $ax + by < c$ and $ax + by \leq c$.

The original version, p'A, presented the student with a step-by-step procedure for drawing the graph of an inequality, as shown on the screen. Then the student was asked to graph an inequality on his own. In addition, one example of an inequality of the form $ax \geq c$ was presented, because equations of the form $ax = c$ had been especially troublesome to the students.

In contrast, the p'B version took the student through a step-by-step slide presentation of the graph of an inequality; the important points were repeated on succeeding examples. For example, in Version p'A, no specific mention was made of the fact that a point which satisfies $ax + by \leq c$ satisfies either $ax + by < c$ or $ax + by = c$, for it was thought that the high ability students would grasp this point. The low ability group, on the other hand, completed an exercise to reinforce the verbal presentation of this point. Also, in Version p'B, the graphing of a straight line was reviewed, for this knowledge was necessary in the graphing of an inequality. The *B* version contained an example of the type ax = c.

The first revision, p''A, consisted mainly in additional explanation and clarification of the two steps in graphing an inequality: graphing a related equation, and determining and shading the area of the inequality. Also more information was presented about the form by = d, and an illustrating slide was added.

In producing Version p''B and p'''B, additional emphasis was also placed on the forms ax = c and by = d, but here an exercise was added to the workbooklets also. The other major revision of the B series was to make clear (with additional text, slides and exercises) that without the correct graph of the related equation, the inequality will be graphed wrong. In addition the graphing of a straight line was reviewed several times.

As in Graphing of Inequalities, coordinate grids, although sometimes omitted here, were included wherever needed on the workbooklets and measures. Since the real problem was whether a student could graph an inequality, items yielding answers to this question were weighted more heavily on the measure.

INEQUALITIES AND THE NUMBER PLANE:

ORIGINAL PROGRAM p' A

INEQUALITIES AND THE NUMBER PLANE - p' A

SCREEN

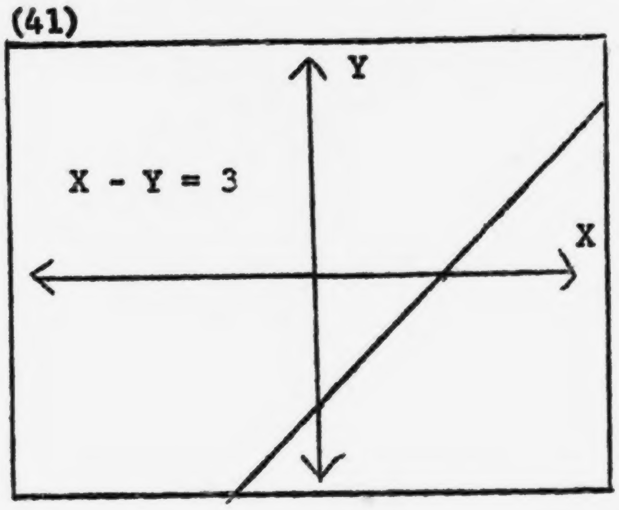
RECORDED SCRIPT

WORKSHEET

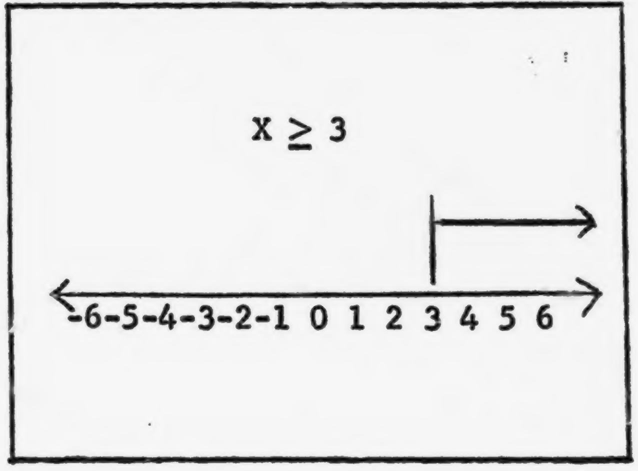
(40)

$X - Y = 3$

(41)



(9)


X ≥ 3
-6-5-4-3-2-1 0 1 2 3 4 5 6

(46)

$X - Y \geq 3$

6

RECORDED SCRIPT

WORKSHEET

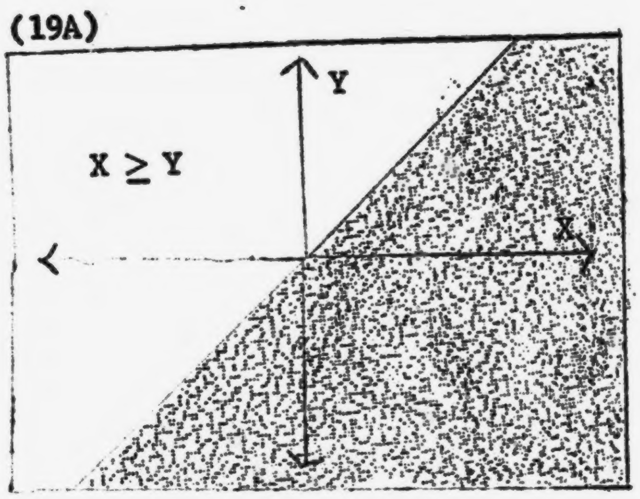
You have worked with equations like $x - y = 3$ (40) and sketched their graphs in the rational number plane. (41) But an equation is only one kind of number statement. (9)

For instance, you have graphed $x \geq 3$ on the number line. Why couldn't we graph a number statement like (46) $x - y \geq 3$ in the number plane? Such a number statement is an inequality and we will now consider the graphing of such statements.

SCREEN

Left Half **Right**

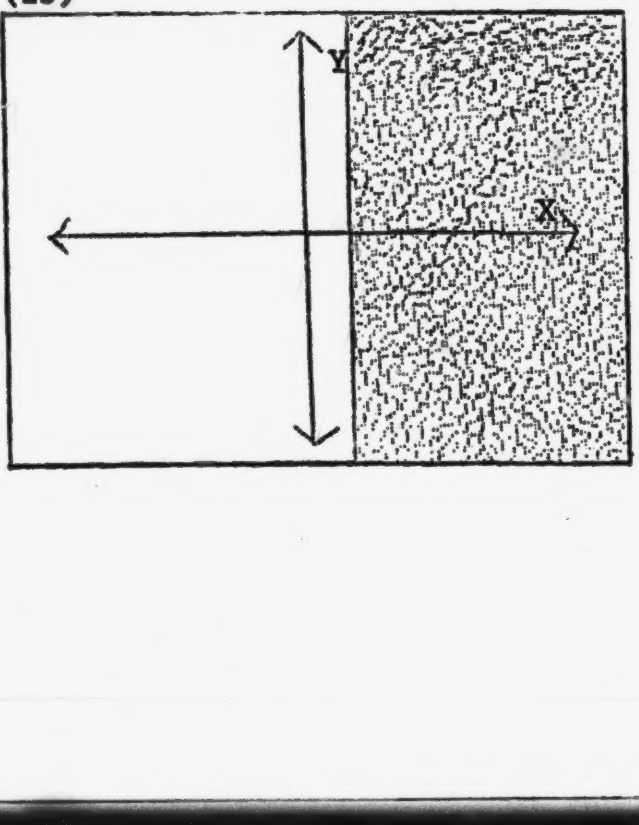
(19A)



'B)

Blank Slide

(23)



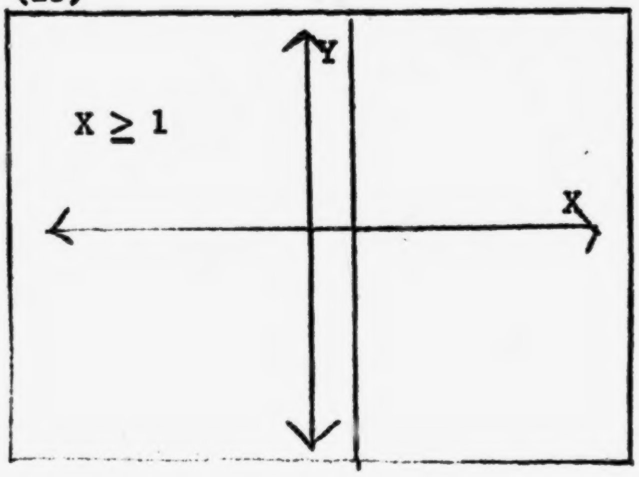
RECORDED SCRIPT	WORKSHEET
(B) Here is the graph of the inequality $x \geq y$. (19A) This graph was drawn by first graphing the line $x = y$, and then shading the area $x > y$.	
To see that any point in the shaded area is a part of the graph, pick a point in the shaded area. Is $x > y$?*	1. Pick a point in the shaded area. (,) Is $x > y$ for your point? ______.
For any point in the shaded area you' picked, you should have found that $x > y$.	
Could a point from the area that is not shaded also satisfy $x > y$? Try one.*	2. Pick a point from the unshaded area. (,) Is $x > y$ for this point? ______
None of the points in the unshaded area satisfy $x > y$.	
(23) Here is a graph of an inequality. The red line is $x = 1$. What is the number statement which represents the whole graph?*	3. Number statement for the graph on the screen is: ______________________________

SCREEN

Left Half

Right Half

(25)



(47R)

$X \geq 1$

$3 > 1$

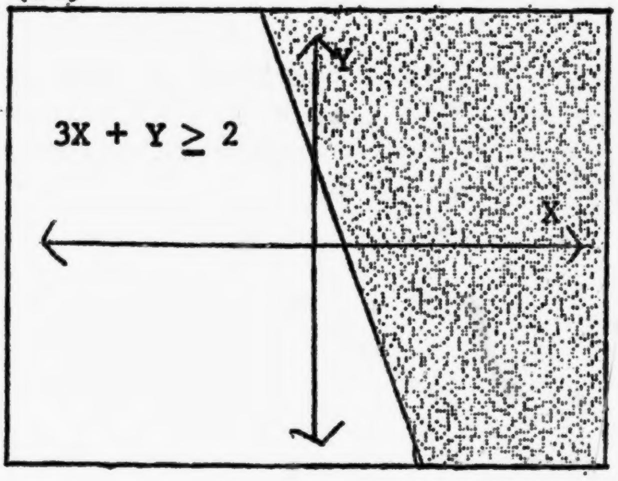
(30)

$3X + Y \geq 2$

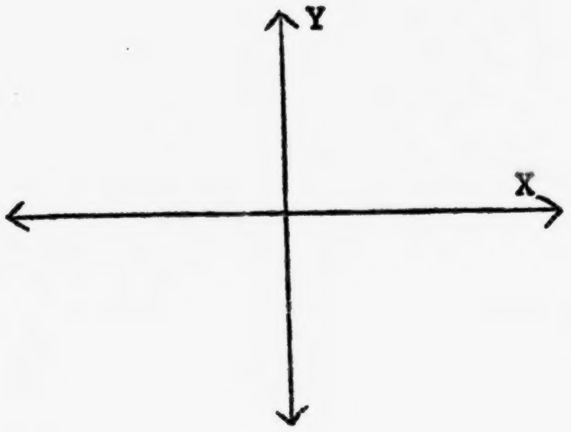
(B)

Blank Slide

(31)



RECORDED SCRIPT	WORKSHEET
(25) This is the graph of $x \geq 1$. (47R) For, say we pick the point $(3,-1)$ from the shaded area. The x coordinate is three, so $x > 1$ and the point satisfies the number statement $x \geq 1$.	
(30) (B) Now graph $3x + y \geq 2$. You might begin by graphing an equation. Then decide on which side of the line the rest of the graph lies, and shade this area.*	4. Graph $3x + y \geq 2$.
(31) Here is $3x + y \geq 2$.	



SCREEN

Left Half

(35R)

Graph

$Y \geq |X|$

Start with

$Y = |X|$

Right Half

(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
(35R) (B) An interesting problem for you to think about is how the graph of $y \geq |x|$ will look. As a hint, start with the graph of $y = |x|$.	

INEQUALITIES AND THE NUMBER PLANE:

ORIGINAL PROGRAM p' B

INEQUALITIES AND THE NUMBER PLANE - p' B

SCREEN

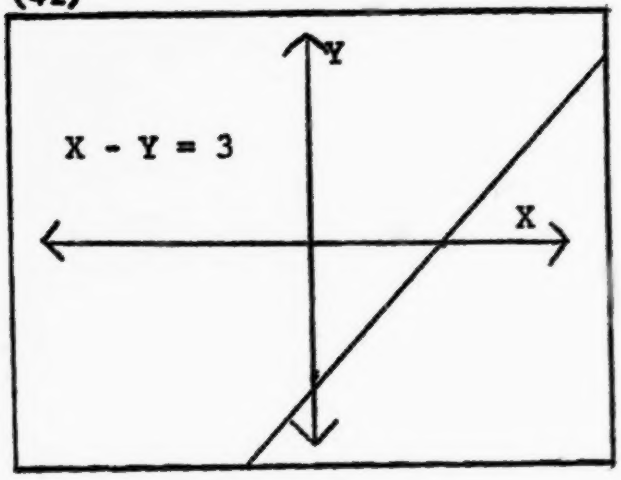
Left Half

Right Half

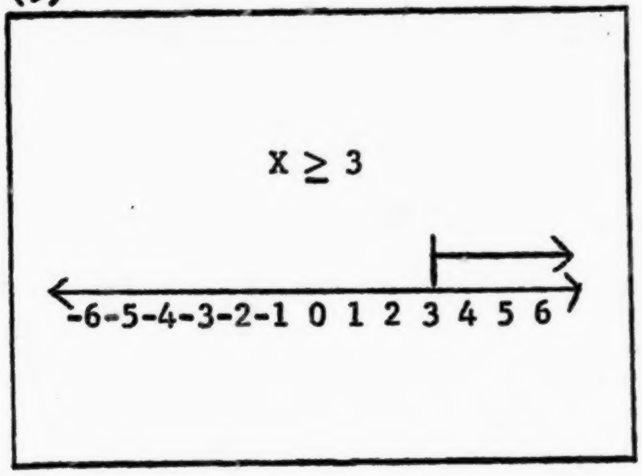
(40)

X - Y = 3

(41)



(9)


$X \geq 3$
-6-5-4-3-2-1 0 1 2 3 4 5 6

(46)

$X - Y \geq 3$

(10A)

$X \geq Y$

(B)

Blank Slide

RECORDED SCRIPT

WORKSHEET

You have worked with equations like $x - y = 3$ (40) and sketched their graphs in the rational number plane. (41) But an equation is only one kind of number statement. (9) For instance, you have graphed $x \geq 3$ on the number line. Why couldn't we graph a number statement like (46) $x - y \geq 3$ in the number plane? Such a number statement is an inequality, and we will now consider the graphing of such statements.

(B) We begin with the inequality $x \geq y$. (10A)

SCREEN

Left Half **Right Half**

(11)

X = Y

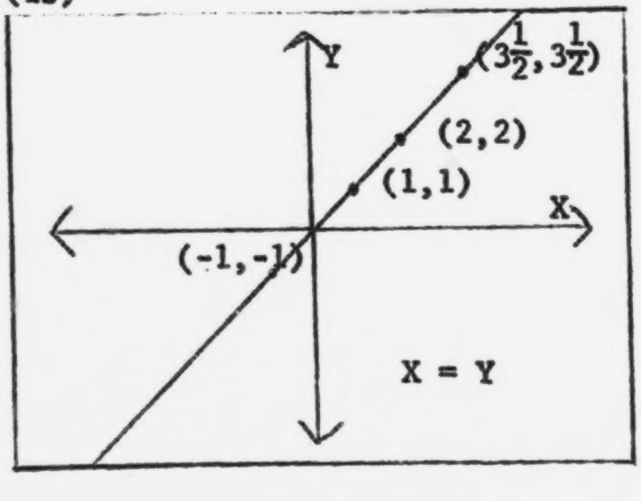
(44)

1) Find three number pairs which satisfy the equation.

2) Plot these points.

3) Draw the straight line through them.

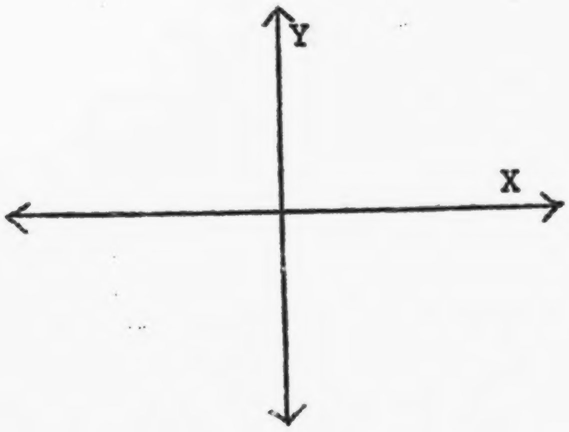
(13)


Y
X
$(3\frac{1}{2}, 3\frac{1}{2})$
(2,2)
(1,1)
(-1,-1)
X = Y

(B)

Blank Slide

RECORDED SCRIPT	WORKSHEET
The best way to graph an inequality is to first graph the (11) related equation, in this case x = y.	
(44) Recall how we graphed such an equation and then graph x = y.*	1. Graph x = y.



(B) The graph of x = y is this straight line (13) which goes diagonally from the lower left hand corner to the upper right hand corner through the origin.

SCREEN

Left Half **Right Half**

(15)

$X \geq Y$

IS THE SAME AS

__________ OR __________

(15A)

$X > Y$

IS THE SAME AS

$X > Y$ OR $X = Y$

(16)

$X > Y$

(,) (,)

(,) (,)

RECORDED SCRIPT	WORKSHEET
(15) Now notice that $x \geq y$ is really made up of two simpler statements. Can you write $x \geq y$ as two simpler statements connected by "or"?*	2. $x \geq y$ is the same as: ____________________ or ____________________
(15A) Recall that $x \geq y$ is just another way of writing $x > y$ or $x = y$.	
Since we have the graph of $x = y$, we now need to find some points which satisfy $x > y$.	
(16) List some number pairs which make $x > y$ a true statement.*	3. List some number pairs which satisfy $x > y$. (,) (,) (,)

SCREEN

Left Half

Right Half

(17)

$X > Y$

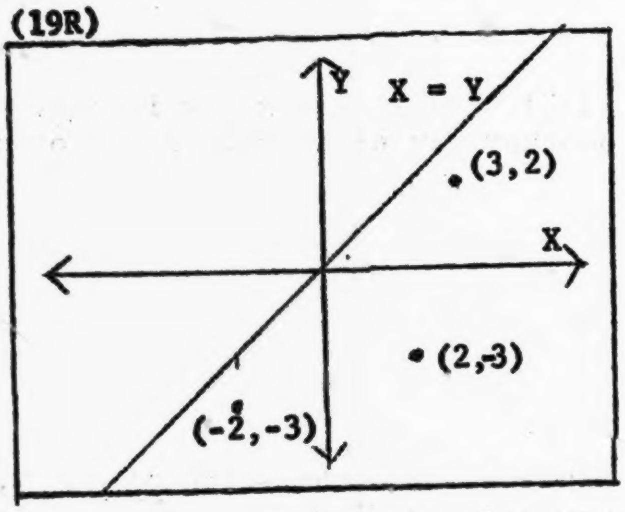
(3,2)

$(\frac{1}{2},\frac{1}{4})$

(-3,-4)

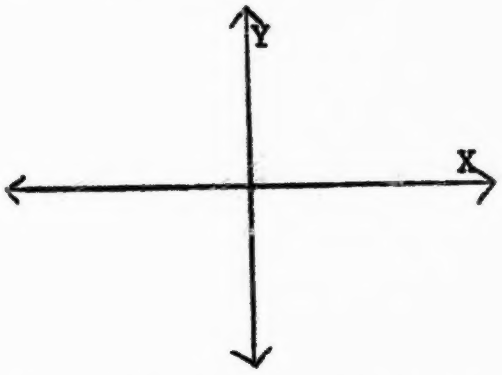
(2,-2)

(19R)



(B)

Blank Slide

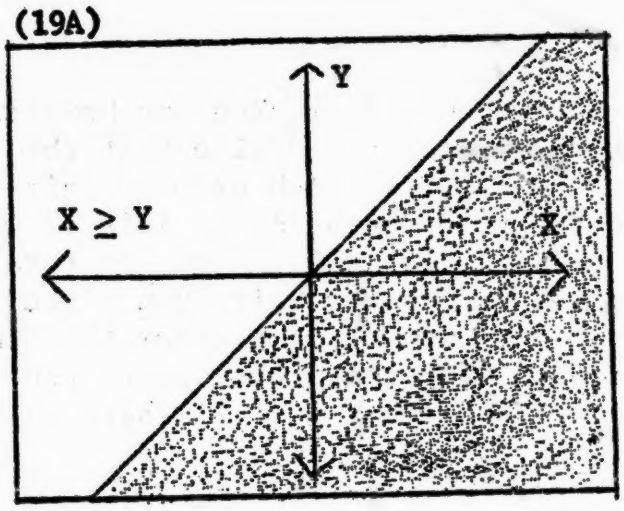
RECORDED SCRIPT	WORKSHEET
Here are some. (17) You may have listed others of course. Notice that the first element of each ordered pair is greater than the second. This is what we mean when we say $x > y$. Be sure that for all the number pairs you listed the first element is greater than the second. Plot the number pairs you found. Note that $x = y$ is already indicated.*	4. Plot your points in (3). 
Where do all the points you plotted lie in relation to $x = y$?*	5. Where do all the points in question lie in relation to $x = y$?
Do all your points lie below (19R) the line $x = y$, like these do? They should.	
(B) Can you find any point below the line which does not satisfy $x > y$?*	6. Is there any point below $x = y$ which does not satisfy $x > y$? ____________________

SCREEN

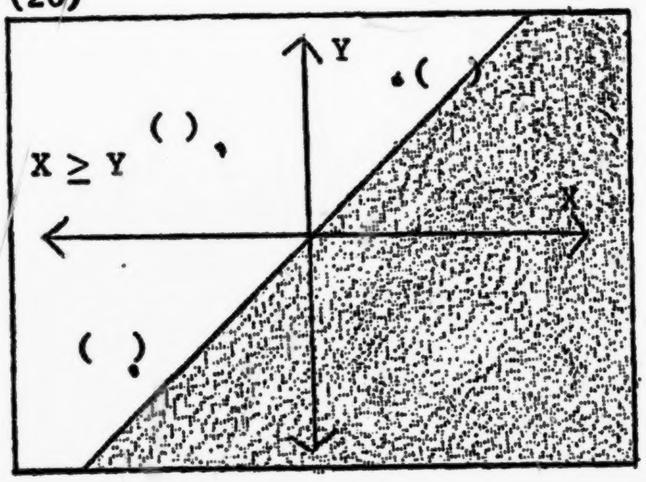
Left Half

Right Half

(19A)



(20)



(B)

Blank Slide

(21)

(2, 3 1/2)

Is X > Y, 2 > 3 1/2 a true statement?

RECORDED SCRIPT	WORKSHEET
You should have found that any point below the line which you tried satisfies $x > y$. So (19A) to indicate the graph of $x > y$ we shade this area. Do this on your graph on question 4.*	
(B) We should also check a point above the line. (20) Pick one of the points indicated here in blue to see if it satisfies $x > y$.*	7. Pick a point. (,) Does it satisfy $x > y$? ________
(21) For instance, one of the points is (2,3 1/2). Here $x < y$ so it does not belong to the graph.	

SCREEN

Left Half

Right Half

(B)

Blank Slide

(26R)

$X \geq -2Y$

(27)

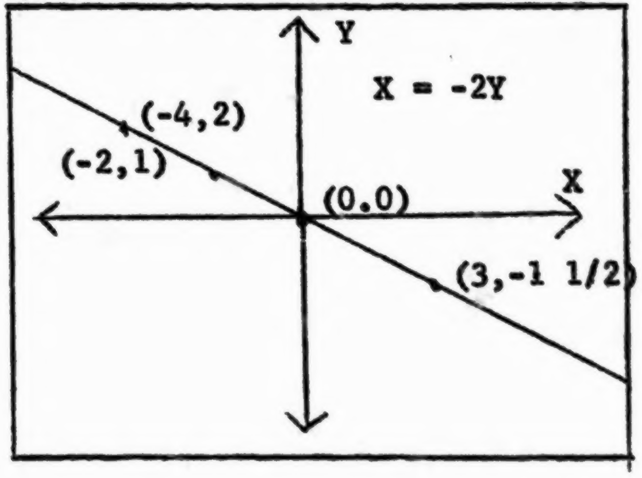
$X = -2Y$

RECORDED SCRIPT	WORKSHEET
(B) Thus the graph of $x \geq y$ is the line $x = y$ plus the red shaded area.	
Now let's graph $x \geq -2y$. (26R) Again we begin by graphing an equation. What equation?*	8. What equation is useful to graph $x \geq -2y$? ________________________
(27) $x = -2y$ is the equation to consider. Graph it.*	9. Graph $x = -2y$.

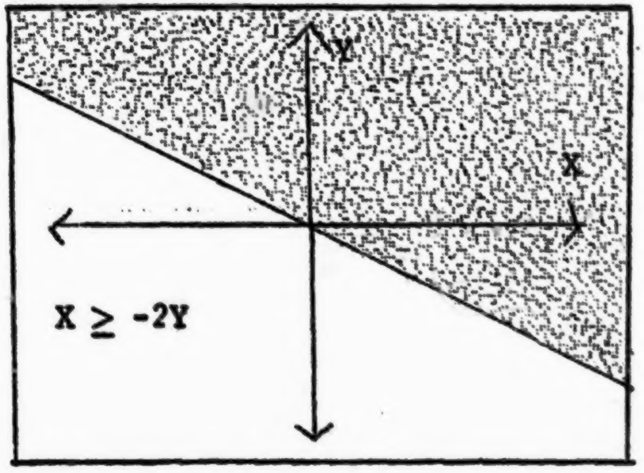
SCREEN

Left Half **Right Half**

(28)



(29R)



(47R)

$X > -2Y$

$1 > -2 \cdot 3$

$1 > -6$

(B)

Blank Slide

RECORDED SCRIPT

WORKSHEET

(28) Here is the graph of $x = -2y$. Now to find the graph of $x > -2y$ try a point above the red line and a point below the red line to see which satisfies $x > -2y$. Shade the area which you think contains points which satisfy $x > -2y$ on the graph in question 9.*

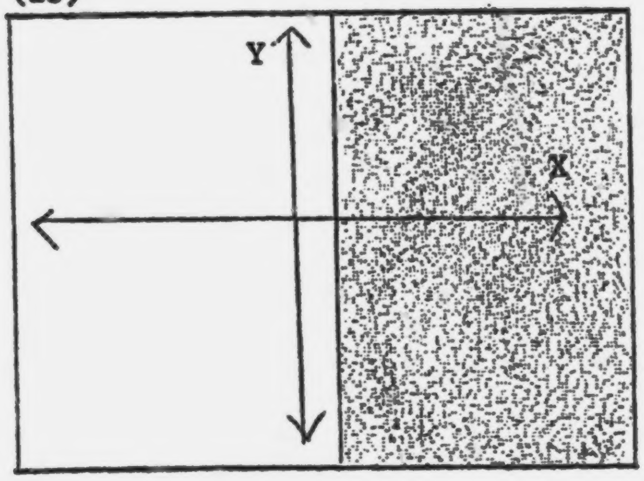
(29R) Points which satisfy $x > -2y$ lie above the red line. (47R) For instance, (1,3) lies above the red line and $1 > -6$.

(B) So, the graph of $x \geq -2y$ is the line plus the shaded area.

SCREEN

Left Half **Right Half**

(23)

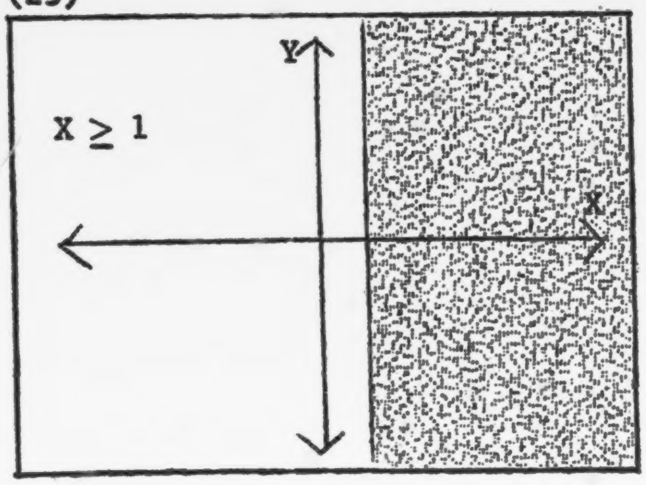


(48)

$X \leq 1$

$X \geq 1$

(25)



(49)

(3,-2)

$3 > 1$

Graph is $X \geq 1$

RECORDED SCRIPTS

(23) Here is the graph of an inequality. You will recall that the equation of the red line is $x = 1$. What inequality is this the graph of?

(48) There are two possibilities, $x \leq 1$ and $x \geq 1$. Try a point in the shaded area and see which inequality is satisfied.*

(25) This is the graph of $x \geq 1$. (49) Consider the point (3,-2) in the shaded area. Certainly $3 > 1$, so it satisfies the inequality $x \geq 1$.

WORKSHEET

10. Pick a point in the shaded area (,)

Does it satisfy $x \leq 1$? ________

Does it satisfy $x \geq 1$? ________

SCREEN

Left Half

(30)

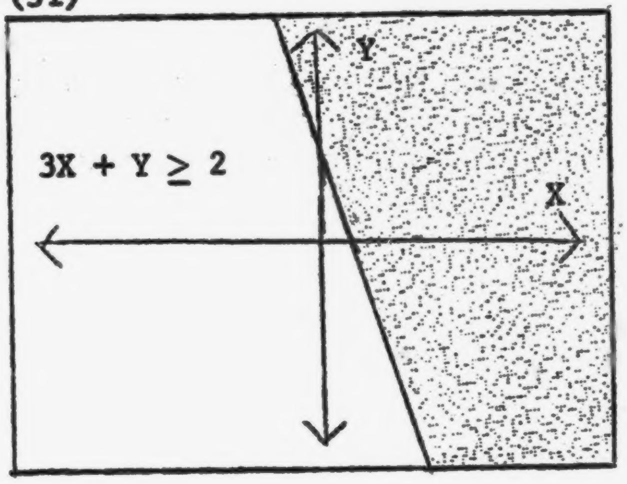
$3X + Y \geq 2$

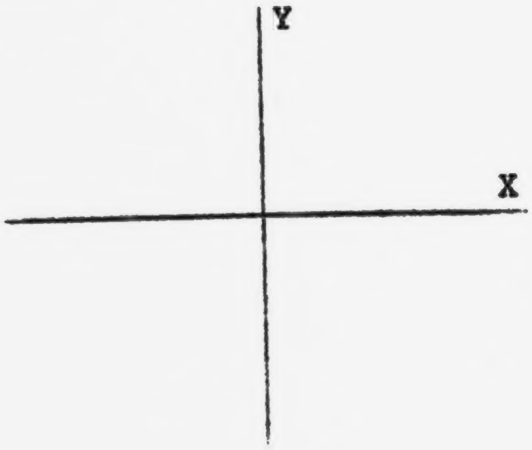
Right Half

(B)

Blank Slide

(31)

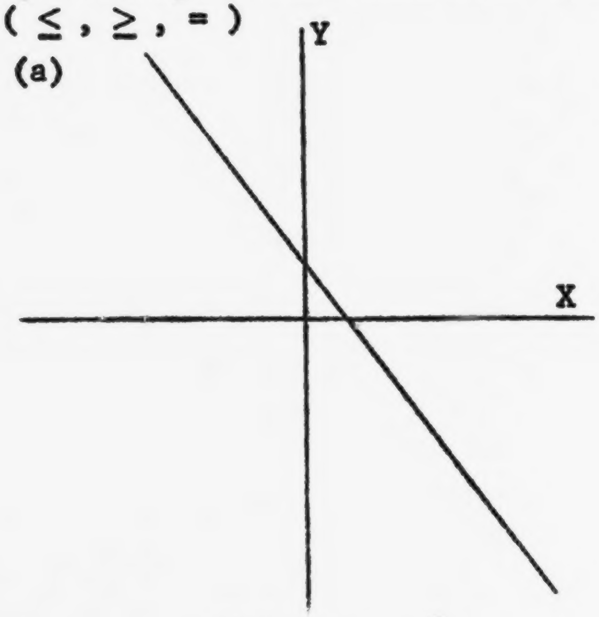


RECORDED SCRIPT	WORKSHEET
(30) Finally, graph $3x + y \geq 2$. First graph the line, then shade the area of the inequality.*	11. Graph $3x + y \geq 2$. 
(31) Here is the result.	

INEQUALITIES AND THE NUMBER PLANE: MEASURE Y_1

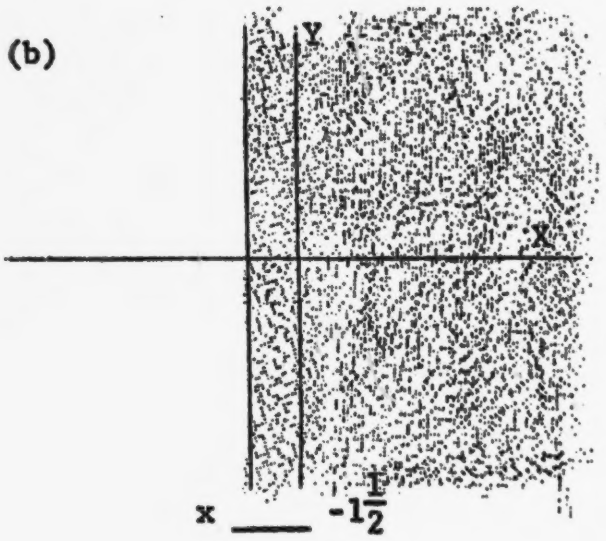
	ANSWERS
1. Break the number statement $3x + 11y \geq 7$ up into an equation and an inequality.	1. $3x + 11y > 7$ or $3x + 11y = 7$
2. Complete the equation of each of these graphs by filling in the blank with the correct sign. ($\leq$, $\geq$, $=$)	2. (a) $x + y = 1$ (b) $x > -1\frac{1}{2}$

(a)



$x + y$ ____ 1

(b)



x ____ $-1\frac{1}{2}$

	ANSWERS
3. Graph each equation	
a.) $y > 3$	
b.) $2x + y \geq 8$	
4. a.) Does (3,6) satisfy $4x + 5y \geq 40$?	4. a.) Yes
b.) Does (-2,12) satisfy $4x + 5y \geq 42$?	b.) Yes

Point distribution: 1-1, 2-2, 3-5, 4-2.
Total: 10

APPENDICES

Appendix A

Table 28. Raw data* for Experiment 1 (Scientific Notation)

GROUP	Ability Level	Version	ADMINISTRATION		
			1	2	3
I	H	A	10, 10, 4	10, 10, 10	10, 8, 9
		B	8, 4, 3, 1	10, 10, 4, 8	10, 10, 4, 10
	L	A	2, 3, 3, 2	3, 9, 8, 1	7, 1, 9, 9
		B	1, 0, 10, 4	5, 1, 9	10, 9, 10, 10
II	H	A		3, 1, 1, 5	6, 3, 4, 5
		B		10, 4, 2	10, 7, 10, 10
	L	A		3, 3, 0, 2	6, 1, 0, 1
		B		3, 5, 5	9, 9, 9, 5
III	H	A	9, 9, 4, 1		2, 10, 1
		B	2, 0, 10		9, 10, 10, 9
	L	A	1, 1, 1		2, 5, 1, 1
		B	0, 3, 3, 10		10, 5, 4
IV	H	A		10, 10, 9	10, 9, 10
		B		9, 7, 10	1, 10
	L	A		1, 3, 3	2, 4
		B		3, 1, 2, 3	10, 6, 10

*Each number represents the points awarded for correct responses on the appropriate measure (Y_1, Y_2, Y_3).

Appendix A (continued)

Table 29. Raw data* for Experiment 2 (System Without Numbers)

GROUP	Ability Level	Version	ADMINISTRATION		
			1	2	3
I	H	A	10, 10, 8	10, 9.5, 5	9, 7.5
		B	9, 10, 8	10, 9.5, 9.5	10, 6.5
	L	A	9, 4, 9, 4	6.5	6.5, 8.5, 2.5
		B	9, 7, 9	4.5, 8.5, 9.5	6.5, 4, 6.5
II	H	A		6, 6, 10, 8	7, 5.5
		B		9, 7, 9	8, 6.5
	L	A		3, 7, 10, 1	3.5, 6.5, 6.5, 3
		B		8, 6, 6	8.5, 5.5
III	H	A	9, 6, 6		9, 9.5, 3.5
		B	5, 10, 10		7, 10
	L	A	4, 6		2.5, 5.5, 3.5
		B	9, 7, 9, 6		
IV	H	A		8, 10, 9, 6	9.5, 7.5, 8
		B		10, 5	9, 9.5
	L	A		3, 7, 5, 6	7, 7, 9.5
		B		9, 9, 8, 6	9, 10, 8.5

*Each number represents the points awarded for correct responses on the appropriate measure (Y_1, Y_2, Y_3).

Appendix A (continued)

Table 30. Raw data* for Experiment 3 (Graphing of Inequalities)

GROUP	Ability Level	Version	ADMINISTRATION		
			1	2	3
I	H	A	5, 4, 6.5, 2.5	9, 5	9, 9, 6
		B	.5, 2.5	4, 6.5, 10, 7.5	2, 9, 10, 7
	L	A	2, 2.5, 2.5	6.5, 4, 9, 4	9, 4, 8.5, 6
		B	2.5, 5, 2.5, 6.25	6, 9, 4.5, 9.5	5, 7, 4, 10
II	H	A		2, 4, .5, 7.5	5, 4
		B		10, 9, 0, 6	8, 10, 9
	L	A		2.5, 1.5, .5	4.5, 1, 3
		B		2.5, .5, 1	7, 6, 2
III	H	A	2, 1		9, 5, 10
		B	2.5, 1.5, 9		2.5
	L	A	4.5, 1, 2, 2.5		5, 10
		B	4, 3, 1		4
IV	H	A		1.5, 2.5, 1.5, 8.5	7, 9, 10
		B		4.5, 1.5, 3.5, 1	9, 7, 8, 7
	L	A		.5, 0, 2.5, 3	2.5, 3, 2.5
		B		2.5, 1.5, 1	5, 3, .5

*Each number represents the points awarded for correct responses on the appropriate Measure (Y_1, Y_2, Y_3).

Appendix A (continued)

Table 31. Raw data* for Experiment 4 (Inequalities and the Number Plane)

GROUP	Ability Level	Version	ADMINISTRATION		
			1	2	3**
I	H	A	5.5, 3.5, 9.5	5.5, 9	
		B	4.5, 10, 6, 5	4.5, 10	
	L	A	2.5, 2	1, 1	
		B	4.5, 4, 6.5	6.5, 4, 2	
II	H	A		5.5, 5.5, 10	
		B		5.5, 10, 3	
	L	A		2, 7.5	
		B		4, 4.5, 3	
III	H	A	8, 4, 9		3.5
		B	6.5, 7.5, 10, 5.5		2.5
	L	A	2, 0		1
		B	3, 3		9
IV	H	A		9, 2	
		B		7.5, 7, 9	
	L	A		5.5	
		B		2, 5.5, 9	

*Each number represents the points awarded for correct responses on the appropriate measure (Y_1, Y_2, Y_3)

**Administration 3 was not completed.

Appendix B

Table 32. Results of the analysis of variance for Experiment 1 (Scientific Notation): first administration of Measure Y_1

Source of Variation	SS	df	MS	F
Ability level	41.286	1	41.286	3.535[a]
Program	.571	1	.571	.038
Ability level X Program	41.286	1	41.286	3.535[a]
Error	280.286	24	11.6719	
Total	363.429	27		

[a] significant at the .10 level

Table 33. Results of the analysis of variance for Experiment 1 (Scientific Notation): second administration of Measure Y_1

Source of Variation	SS	df[1]	MS	F
Ability level	99.076	1	99.076	12.672[a]
Program	12.449	1	12.449	1.592
Ability level X Program	.779	1	.779	.100
Error	179.819	23	7.818	
Total	292.123	26		

[a] significant at the .01 level

[1] In addition to the usual procedure of subtracting one df for the grand mean, one df was subtracted when a cell entry was estimated by using the mean of the cell.

Appendix B (continued)

Table 34. Results of the analysis of variance for Experiment 2 (System without Numbers): First administration of Measure Y_1

Source of Variation	SS	df	MS	F
Ability level	9.375	1	9.375	2.622
Program	12.042	1	12.042	3.369[a]
Ability level X Program	5.042	1	5.042	1.410
Error	71.500	20	3.575	
Total	97.959	23		

[a] significant at the .10 level

Table 35. Results of the analysis of variance for Experiment 2 (System without Numbers): second administration of Measure Y_1

Source of Variation	SS	df[1]	MS	F
Ability level	14.000	1	14.000	2.963
Program	8.556	1	8.556	1.811
Ability level X Program	2.451	1	2.451	.519
Error	89.759	19	4.724	
Total	114.766	22		

[1] In addition to the usual procedure of subtracting one df for the grand mean, one df was subtracted when a cell entry was estimated by using the mean of the cell.

Table 36. Results of the analysis of variance for Experiment 3 (Graphing of Inequalities): first administration of Measure Y_1

Source of Variation	SS	df[1]	MS	F
Ability level	.023	1	.023	.005
Program	1.148	1	1.148	.253
Ability level X Program	6.253	1	6.253	1.375
Error	86.383	19	4.546	
Total	93.807	22		

[1] In addition to the usual procedure of subtracting one df for the grand mean, one df was subtracted when a cell entry was estimated by using the mean of the cell.

Table 37. Results of the analysis of variance for Experiment 3 (Graphing of Inequalities): second administration of Measure Y_1

Source of Variation	SS	df[1]	MS	F
Ability level	48.758	1	48.758	7.222[a]
Program	1.758	1	1.758	.260
Ability level X Program	1.758	1	1.758	.260
Error	168.779	25	6.751	
Total	221.053	28		

[a] significant at the .05 level

[1] In addition to the usual procedure of subtracting one df for the grand mean, one df was subtracted when a cell entry was estimated by using the mean of the cell.

Table 38. Results of the analysis of variance for Experiment 4 (*Inequalities and the Number Plane*): first administration of Measure Y_1

Source of Variation	SS	df[1]	MS	*F*
Ability level	78.013	1	78.013	21.955[a]
Program	4.638	1	4.638	1.305
Ability level X Program	15.188	1	15.188	4.274[b]
Error	53.300	15	3.553	
Total	151.139	18		

[a] significant at the .001 level
[b] significant at the .10 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted when a cell entry was estimated by using the mean of the cell.

Table 39. Results of the analysis of variance for Experiment 4 (*Inequalities and the Number Plane*): second administration of Measure Y_1

Source of Variation	SS	df[1]	MS	*F*
Ability level	73.153	1	73.153	13.017[a]
Program	13.203	1	13.203	2.350
Ability level X Program	5.253	1	5.253	.935
Error	84.298	15	5.620	
Total	175.907	18		

[a] significant at the .01 level

[1] In addition to the usual procedure of subtracting one *df* for the grand mean, one *df* was subtracted when a cell entry was estimated by using the mean of the cell.

Appendix C

Resume of the Report of the Electronics Technician Concerning Modifications to the Edex Teaching System

The problems encountered at the outset of this project involved the lack of independent projector control and the lack of mobility of control in the Edex system. The Edex console did not allow simultaneous projection of equipment; rather, the Edex operator was required to use projectors alternately. The system further required that a teacher or operator be present at the console lectern if he wanted to operate the equipment manually. After an examination of the schematic drawings and a trip to the school plant to examine the equipment, the following electrical changes were made to solve the aforementioned problems.

Examination of the equipment and the Edex schemata revealed that both slide changers could be triggered simultaneously or in any random fashion desired. However, the projector lamps would operate only on alternate pulses. This problem was overcome by fitting a special trip cord to an additional Carousel projector; this cord was attached to the existing projector junction box in the projection area. By using the film strip projector trip for an additional Carousel trip and by plugging the Carousel into a separate power source for the projector lamp, the slide change functions of the existing Edex equipment could be used to operate two slide projectors on the screen simultaneously or in any random fashion of operation.

The second problem, mobility of control, was approached by manufacturing a long extension cord with a pair of double-pole single-throw switches attached to one end and a Jones connector fastened to the other end. This cord was connected to a receptacle added to and mounted in the Edex control console. A parallel circuit was wired from this receptacle into the Edex teacher-control buttons, thus giving the teacher remote operation of the Edex equipment.

These modifications of the existing equipment allowed the necessary projector functions to complete the present project. However, the modifications were not extensive enough to allow full-scale multimedia presentations, *i.e.*, the operation of three projectors in any random fashion programed into the equipment by the operator. The best method for overcoming these limitations would be to incorporate into the system some form of programing mechanism, preferably a multi-channel control device. There are several such devices on the market which could be adapted for use with the Edex and which could provide up to eight control channels.

Appendix D

Table 40. Experiment 1 - Analysis 1: Results of the Newman-Keuls analysis of the Ability level X Program interaction

Critical Ratio	Mean 1 6.413	Mean 2 5.619	Mean 3 3.571	Mean 4 2.000	
2.38		(2) .79	(3) 2.84	(4) 4.41*	Mean 1 6.413
2.86			(2) 2.04	(3) 3.61*	Mean 2 5.619
3.15				(2) 1.57	Mean 3 3.571
					Mean 4 2.000

*significant at the .05 level

Table 41. Experiment 1 - Analysis 2: Results of the Newman-Keuls analysis of the Ability level X Program interaction

Critical Ratio	Mean 1 53.542	Mean 2 52.226	Mean 3 49.286	Mean 4 43.239	
4.17		(2) 1.32	(3) 4.26	(4) 10.30*	Mean 1 53.542
5.06			(2) 2.94	(3) 8.99*	Mean 2 52.226
5.60				(2) 6.05*	Mean 3 49.286
					Mean 4 43.239

*significant at the .05 level

Appendix D (continued)

Table 42. Experiment 1 - Analysis 3: Results of the Newman-Keuls analysis of the Ability level X Program interaction

Critical Ratio	Mean 1 54.214	Mean 2 54.186	Mean 3 50.000	Mean 4 46.471	
8.50		(2) .028	(3) 4.21	(4) 13.74*	Mean 1 54.214
10.31			(2) 4.19	(3) 13.72*	Mean 2 54.186
11.40				(2) 9.53	Mean 3 50.000
					Mean 4 40.471

*significant at the .05 level

Table 43. Experiment 1 - Analysis 3A: Results of the Newman-Keuls analysis of the Ability level X Program interaction

Critical Ratio	Mean 1 54.614	Mean 2 53.557	Mean 3 49.114	Mean 4 41.085	
6.38		(2) 1.06	(3) 5.50	(4) 13.53	Mean 1 54.614
7.67			(2) 4.44	(3) 12.47*	Mean 2 54.557
8.45				(2) 8.03	Mean 3 49.114
					Mean 4 41.085

*significant at the .05 level

Appendix D (continued)

Table 44. Experiment 1 - Analysis 4: Results of the Newman-Keuls analysis of the Program X Group interaction

Critical Ratio	Mean 1 9.125	Mean 2 8.625	Mean 3 7.750	Mean 4 3.2500	
2.36		(2) .50	(3) 1.38	(4) 5.88*	Mean 1 9.125
2.86			(2) .88	(3) 5.38*	Mean 2 8.625
3.17				(2) 4.50*	Mean 3 7.750
					Mean 4 3.250

*significant at the .05 level

Table 45. Experiment 1 - Analysis 4: Results of the Newman-Keuls analysis of the Ability level X Program interaction

Critical Ratio	Mean 1 8.889	Mean 2 8.889	Mean 3 6.750	Mean 4 4.250	
2.36		(2) 0.00	(3) 2.14	(4) 4.64*	Mean 1 8.889
2.86			(2) 2.14	(3) 4.64*	Mean 2 8.889
3.17				(2) 2.50*	Mean 3 6.750
					Mean 4 4.250

*significant at the .05 level

Appendix D (continued)

Table 46. Experiment 2 - Analysis 1: Results of the Newman-Keuls analysis of the Ability level X Program interaction

Critical Ratio	Mean 1 8.400	Mean 2 8.000	Mean 3 7.714	Mean 4 5.571	
1.49		(2) .40	(3) .69	(4) 2.83*	Mean 1 8.400
1.79			(2) .29	(3) 2.43*	Mean 2 8.000
1.97				(2) 2.14*	Mean 3 7.714
					Mean 4 5.571

*significant at the .05 level

Table 47. Experiment 3 - Analysis 1: Results of the Newman-Keuls analysis of the Ability level X Program interaction

Critical Ratio	Mean 1 3.988	Mean 2 3.500	Mean 3 2.470	Mean 4 1.179	
1.80		(2) .49	(3) 1.52	(4) 2.81*	Mean 1 3.988
2.17			(2) 1.03	(3) 2.32*	Mean 2 3.500
2.39				(2) 1.29	Mean 3 2.470
					Mean 4 1.179

*significant at the .05 level

Appendix D (continued)

Table 48. Experiment 3 - Analysis 4: Results of the Newman-Keuls analysis of the Program X Group interaction

Critical Ratio	Mean 1 7.083	Mean 2 7.000	Mean 3 6.667	Mean 4 3.667	
3.15		(2) .08	(3) .42	(4) 3.42	Mean 1 7.083
3.85			(2) .33	(3) 3.33	Mean 2 7.000
4.27				(2) 3.00	Mean 3 6.667
					Mean 4 3.667

*significant at the .05 level

Table 49. Experiment 4 - Analysis 2: Results of the Newman-Keuls analysis of the Ability level X Program interaction

Critical Ratio	Mean 1 54.440	Mean 2 54.080	Mean 3 47.300	Mean 4 29.980	
15.06		(2) .36	(3) 7.14	(4) 24.46*	Mean 1 54.440
18.39			(2) 6.78	(3) 24.10*	Mean 2 54.080
20.43				(2) 17.32*	Mean 3 47.300
					Mean 4 29.980

*significant at the .05 level

Appendix E

Table 50. STEP mathematics scores* for subjects in Experiments 1 and 2

Subject	Score	Subject	Score	Subject	Score
1	95	22	80	43	52
2	93	23	76	44	52
3	93	24	76	45	52
4	91	25	76	46	52
5	91	26	73	47	48
6	91	27	73	48	44
7	89	28	73	49	44
8	89	29	73	50	44
9	86	30	73	51	44
10	86	31	69	52	37
11	83	32	65	53	37
12	83	33	65	54	29
13	83	34	61	55	25
14	83	35	61	56	25
15	83	36	61	57	22
16	80	37	61	58	22
17	80	38	61	59	22
18	80	39	57	60	19
19	80	40	57	61	13
20	80	41	57	62	11
21	80	42	52		

*Scores presented are percentile scores.

Appendix E (continued)

Table 51. STEP mathematics scores* for subjects in Experiments 3 and 4

Subject	Score	Subject	Score	Subject	Score
1	99	23	80	45	54**
2	98	24	80	46	54**
3	97	25	80	47	52
4	93	26	80	48	52
5	93	27	76**	49	50**
6	93	28	76**	50	50**
7	92**	29	76	51	50**
8	91**	30	76	52	50**
9	89**	31	73	53	44
10	89	32	73	54	44
11	89	33	73	55	44
12	87**	34	69	56	40
13	87**	35	69	57	40
14	87**	36	65	58	40
15	87**	37	65	59	37
16	86	38	65	60	36**
17	86	39	63**	61	33
18	86	40	61	62	33
19	86	41	59**	63	25
20	83	42	57	64	22
21	83	43	57		
22	83	44	57		

*Scores presented are percentile scores.

**The subject achieving this score was a junior.

Appendix E (continued)

Appendix E (continued)

Table 52. STEP Mathematics Section data[*]

Class (Number of subjects)	Mean	Median	Range
Freshmen (62)	62.79	67.00	11-95
Sophomores (46)	67.70	72.83	22-99
Juniors (18)	66.20	68.50	36-92
Sophomores and Juniors combined (64)	67.28	72.83	22-99

[*] The data presented are percentiles.

Appendix F

Text and tabulation of a questionnaire given to the Wisconsin Heights High School faculty concerning the Edex Teaching System

A multimedia expert has suggested that certain minor alterations to our existing Edex equipment might make it much more versatile. We would like to find out what could be done to make the Edex equipment in the HLG more useful to you. Would you please fill in this questionnaire and return it to the office by Thursday, March 3. Check as many items in each category as apply to you.

Number of Responses[*]		
	I.	Present use of the Edex. This semester I have used or am planning to use the Edex equipment in the following way(s):
10	a.	Slide projector controls.
9	b.	Film strip projector controls.
10	c.	Motion picture projector controls.
5	d.	Tape recorder to play ordinary tape recordings.
2	e.	Tape recorder to turn projectors on and off.
4	f.	Response buttons and meters or score accumulators.
0	g.	Other uses you make of Edex (Specify) ____________
	II.	Problems experienced in using the Edex.
7	a.	Insufficient knowledge as to how to operate it.
6	b.	Insufficient knowledge as to its possible uses.
0	c.	Too complex to be useful.
0	d.	Does not do easily the kinds of things you would like it to do (Specify) ____________
2	e.	No need for it in your instructional program.
2	f.	Other problems in using the equipment (Specify) No time to set up equipment because of room use. Teachers do not leave equipment in operative order.
3	g.	You prefer to use a projector in the rear of the HLG, rather than an Edex projector. (Please explain why) Not enough practice on Edex. Equipment not always operative. No time to program.

---

[*] 17 of approximately 30 faculty members returned the questionnaire. However, two of these checked II e., and two additional teachers stated that they did not use the HLG, the room in which the Edex was located. These four teachers responded to none of the rest of the questionnaire, so essentially these numbers represent the responses of 13 teachers.

Appendix F (continued)

III. Modes of operation the Edex should be capable of.

8 a. Slide and filmstrip projectors on at the same time.
6 b. Two slide projectors on at the same time.
2 c. Two filmstrip projectors on at the same time.
2 d. Movie projector and slide projector on at the same time.
2 e. Movie projector and filmstrip projector on at the same time.
2 f. Some combination of three projectors on at one time. Specify: One slide and two filmstrip projectors
10 g. In addition to the present advance button for filmstrips or slides, a reverse button.
4 h. Random access to slides.
11 i. Focus control for slide projector at Edex console.
8 j. More student response stations.
6 k. A way to obtain permanent record of student responses.
1 l. Remote operation of certain functions of the Edex. Specify which functions: On-off and Change Slides.
6 m. Better orientation of the Edex console in the HLG. Give suggestions below.
4 n. Standard programmed tapes to perform certain sequences of operations.
0 o. Other (Specify). ____________

IV. Suggestions. Please give any criticisms or suggestions you have concerning the Edex machine and associated equipment. Comment further on any of the above questions if you like.* Thank you!

---

*Other criticisms and suggestions included:

1. Students have difficulty seeing the visuals.
2. Console needs to be more flexible in positioning and wires concealed.
3. Console lacks storage space to keep "junk" off the top.
4. Room light control needs to be located at the console.
5. Include automatic screens for the overhead.
6. Permanently anchor the seats and include more seats in the room.